ESTIMASI: Journal of Statistics and Its Application
Vol. 5, No. 1, Januari, 2024, Hal. 250-265
DOI: https://doi.org/10.20956/ejsa.v5i2.31904

Modeling Exchange Rate of Naira to Euro with the APLSTAR-GARCH model

Benjamin .A. Effiong[1], Emmanuel .W. Okereke[2*], Chukwuemeka .O. Omekara[3], Chigozie .K. Acha[4] and Emmanuel .A. Akpan[5]
[1]Department of Statistics, Akwa Ibom State Polytechnic, Ikot Osurua. Akwa Ibom State.
[234]Department of Statistics, Michael Okpara University of Agriculture, Umudike, Abia State.
[5]Department of Basic Sciences, Federal College of Medical Laboratory Science and Technology, Jos, Plateau State.
***Corresponding author, email: okereke.emmanuel@mouau.edu.ng**

Abstract

Application of the asymmetric power logistic smooth transition autoregressive (APLSTAR) model proposed by [1] to naira/Euro exchange rate spanning from January, 2006 to April, 2021, which is a nonlinear macroeconomic time series was considered. The APLSTAR model was justifiably fitted to the series and the fit of the APLSTAR model compared with the fits of the competing models revealed that the APLSTAR model fits the data exchange rate of naira to Euro better than the other asymmetric STAR models. Lagrange Multiplier tests for autoregressive conditional heteroscedastic (ARCH) effects were carried out and there was no substantial evidence to reject the presence of ARCH effects in the set of residuals used. Hence, we compared hybrid smooth transition autoregressive-generalized ARCH (STAR-GARCH) models using model evaluation criteria. On balance, the APLSTAR-GARCH (0, 1) model outperforms the other models under consideration.

***Keywords**: ARCH effects, Model evaluation criteria, Nonlinear macroeconomic time series, Parameter estimate, Smooth transition autoregressive model*

1. Pendahuluan

Most of the macroeconomic time series (METS) often exhibit behaviour that changes suddenly. According to [2], METS are most often the results of decisions made by a large number of economic agents. Sometimes, when examined graphically, the values of METS slowly rise to a peak and then quickly decline to a trough or the values may rise quickly to a peak and then slowly decline to a trough causing regime shifts which may be due to a several causes [3, 4]. Hence, the need to model such series with regime switching models such as smooth transition autoregressive (STAR) model. A number of empirical studies have shown that STAR models characterize the dynamics of exchange rates [5, 6, 7, 8, 9, 10, 11, 12].

The performance rating of STAR models depend largely on the transition function and the METS under consideration. The forecast performance of STAR models varies from one series to another and largely determined by transition function of the STAR model. There are situations whereby new transition function with corresponding STAR model is needed to specifically model the behaviour of a METS, hence good forecasts

[1]. The aim of this paper is to compare the forecast performance of asymmetric power logistic smooth transition autoregressive (APLSTAR) model with the existing asymmetric STAR models.

On the other hand, a good number of METS exhibit volatility clustering and other nonlinear features. Fitting only a nonlinear conditional mean models like TAR models, STAR models, etc., to such METS often result in residuals that are not free from autoregressive conditional heteroscesdastic (ARCH) effects. According to [4], a variance that changes over time affect the outcomes of statistical inference about the parameters that describe the dynamics of the level of the series. Hence, the need to forecast not only the level of METS, but also its variance. To model such series, it is necessary to combine both conditional mean model and conditional variance model to capture both regime switching behaviour and heteroscedasticity in the series [13]. The paper compares the efficiency of APLSTAR-Generalized autoregressive conditional heteroscesdastic (APLSTAR-GARCH) model with its constituent models and the competing models.

This paper is further organized as follows: Section 2 covers the material and method. Results and discussion of findings are presented in Section 3, while Section 4 concludes the study.

2. Material dan Metode

2.1 Data

Monthly exchange rate of naira to Euro comprising 184 observations spanning from January, 2006 to April, 2021, extracted from the Central bank of Nigeria (CBN) statistical bulletin are used for empirical analyses after transformation. The transformed data, $y_t = 100(\mathrm{log x_t} - \mathrm{log x_{t-1}})$, where x_t is the exchange rate of naira to Euro.

2.2 Asymmetric STAR models and their corresponding transition functions

A two-regime STAR model for a univariate time series y_t, which is observed at $t = 1-p, 1-(1-p), \dots, -1, 0, 1, \dots, T-1, T$ is given by

$$y_t = \left(\xi_{1,0} + \xi_{1,1}y_{t-1} + \cdots + \xi_{1,\mathrm{p}}y_{t-p}\right)\left(1 - \mathrm{F}(y_{t-d}; \delta, \lambda)\right) + (\xi_{2,0} + \xi_{2,1}y_{t-1} + \cdots + \xi_{2,\mathrm{p}}y_{t-p})\mathrm{F}(y_{t-d}; \delta, \lambda)) + \nu_{t.} \quad (1)$$

(1) can be rewritten as

$$y_t = \boldsymbol{\xi}_1' \boldsymbol{w}_t(1 - \mathrm{F}(y_{t-d}; \delta, \lambda)) + \boldsymbol{\xi}_2' \boldsymbol{w}_t \mathrm{F}(y_{t-d}; \delta, \lambda) + \boldsymbol{\nu}_{t,} \quad (2)$$

where $\boldsymbol{\xi}_\mathrm{i} = (\xi_{\mathrm{i},0}, \xi_{\mathrm{i},1}, \cdots, \xi_{\mathrm{i,p}})'$ for $i = 1,2$, $\boldsymbol{w}_t = (1, y_{t-1}, \dots, y_{t-p})'$. $\mathrm{F}(y_{t-d}; \delta, \lambda)$ is the transition function. The $\nu_t's$ are assumed to be a martingale difference sequence with respect to the history of the time series up to time $t-1$, which is donated as $\Omega_{t-1} = \{y_{t-1}, y_{t-2}, \dots, y_{1-(p-1),} y_{1-p}\}$, $E[\nu_t/\Omega_{t-1}] = 0$, and $E[\nu_t^2/\Omega_{t-1}] = \sigma^2$ [14].

The first-order logistic function (LSTR1) proposed by [2] given by

$$F(y_{t-d};\delta,\lambda) = (1+\exp[-\delta(y_{t-d}-\lambda)])^{-1},\ \delta>0, \quad (3)$$

and the STAR model (2) with (3) is called the logistic STAR (LSTAR) model.
The second-order logistic (LSTR2) function given by

$$F(y_{t-d};\delta,\lambda) = (1+\exp[-\delta(y_{t-d}-\lambda_1)(y_{t-d}-\lambda_2)])^{-1}, \lambda_1 \le \lambda_2, \delta>0, \quad (4)$$

where $\lambda=(\lambda_1,\lambda_2)'$, as proposed by [15].

Another common choice of $F(y_{t-d};\delta,\lambda)$ is the exponential function proposed by [2] given by

$$F(y_{t-d};\delta,\lambda) = 1-\exp[-\delta(y_{t-d}-\lambda)^2],\ \delta>0. \quad (5)$$

Model (2) with (5) is called exponential STAR (ESTAR) model.
The alternative representation of LSTAR model proposed by [16] called error logistic function is given by

$$F(.) = \{1+\exp\left[-\delta(e_t(AR(p))-\lambda\right]\}^{-1}-\frac{1}{2}, \quad (6)$$

where e_t is the error of prediction from the autoregressive process of order p.
Model (6) with (2) is called error logistic smooth transition regressive (ELSTR) model.
Power logistic function is given by

$$F(y_{t-d};\delta,\lambda) = \left\{1+0.5\exp[-\delta\left(y_{t-d}^{i}-\lambda\right)]\right\}^{-2}-\frac{1}{1.5^2}, \delta>0,\ i=1,2, \quad (7)$$

where δ is the slope parameter and λ is the location parameter.
$\frac{1}{1.5^2}$ is subtracted from (7) to ensure that $F(y_{t-d};\delta=0,\lambda)=0$ and it is useful in performing linearity tests. (7) is a transition function possessing a non-zero derivative of order $(2s+1), s=0,1$. Also, $F(y_{t-d};\delta=0,\lambda)=0$ and $\frac{d^k}{d\delta^k}F(\delta)\Big|_{\delta=0} \neq 0$, for $k=1,3$ and $1\le k \le 2s+1$ [1].
The asymmetric PLSTAR (APLSTAR) model is

$$F(y_{t-d};\delta,\lambda) = \{1+0.5\exp[-\delta(y_{t-d}-\lambda)]\}^{-2}-\frac{1}{1.5^2}, \delta>0, \quad (8)$$

Model (7) with (2) is called asymmetric Power logistic STAR (APLSTAR) model.

2.3 Conditional Heteroscedastic Models

According to [3], the autoregressive moving average process of order l and m, $\mu(L)y_t = \mu_0 + \mu(L)\nu_t$ can be written as the sum of a predictable part and a prediction error as

$$y_t = E[y_t|\Omega_{t-1}] + \nu_t, \quad (9)$$

where Ω_{t-1} represents the past information available up to time $t-1$ and ν_t represents the prediction error. The prediction errors ν_t are independent random variables with a constant variance $Var(\nu_t) = \sigma_\nu^2$ that is independent of the past.

The autoregressive conditional heteroscedastic (ARCH) model of order l, denoted ARCH(l) introduced by [17] is given by

$$\nu_t = \sigma_t z_t, \quad (10)$$
$$\sigma_t^2 = \lambda_0 + \lambda_1\nu_{t-1}^2 + \lambda_2\nu_{t-2}^2 + \cdots + \lambda_l\nu_{t-l}^2, \quad (11)$$

where $\sigma_t^2 = \mathrm{Var}[\nu_t|\Omega_{t-1}]$ denote the conditional variance of ν_t, $\{z_t\}$ is a sequence of independent and identically distributed (iid) random variables with mean zero and variance 1, $\lambda_0 > 0$, and $\lambda_i \geq 0$ for $i = 1,2,\dots,l-1$, and $\lambda_l > 0$. The constraint $\sum_{i=1}^{l}\lambda_i < 1$ ensures that ν_t are covariance stationary with finite unconditional variance (σ_ν^2). The basic idea of ARCH models is that the shock ν_t of a return series is serially uncorrelated, but dependent and the dependence of ν_t can be described by a simple quadratic function of its lagged values [18].

The generalized ARCH model of order l and m, denoted GARCH(l,m) model introduced by [19] is given by

$$\sigma_t^2 = \lambda_0 + \sum_{i=1}^{l}\lambda_i\nu_{t-i}^2 + \sum_{j=1}^{m}\beta_j\,\sigma_{t-j}^2\,, \quad (12)$$

where $\lambda_0 > 0$, and $\lambda_i \geq 0$ for $i = 1,2,\dots,l-1$, and $\lambda_l > 0$, $\beta_j \geq 0$ for $j = 1,2,\dots,m-1$, and $\beta_m > 0$. In practice, z_t is often assumed to follow the standard normal (norm) or a standardized Student-*t* distribution (std) or a skewed Student-*t* distribution (sstd) or a generalized error distribution (ged) or a skewed generalized error distribution (sged), etc.

2.3.1 Test for ARCH/GARCH Effects

Lagrange multiplier (score) test proposed by [20] is used to test the null hypothesis $H_0 = \lambda_i = 0, i = 1,2,\dots,l$ (no ARRCH effects). The score statistic denoted by Λ is given by

$$\Lambda = TR^2, \quad (13)$$

where T is the sample size and R^2 is the coefficient of determination computed from the following auxiliary regression equation:

$$\hat{v}_t^2 = \lambda_0 + \lambda_1\hat{v}_{t-1}^2 + \lambda_2\hat{v}_{t-2}^2 + \cdots + \lambda_l\hat{v}_{t-l}^2 + \eta_t. \tag{14}$$

The score test has an asymptotic chi-square distribution with l degrees of freedom under the null hypothesis of no ARCH effects. If the ARCH effect is found to be significant, the ARCH order is determined using the PACF of $\hat{v}_t^2$.

2.4 Specification, Estimation and Evaluation of STAR-GARCH models

Specification of STAR models is based on Terasvirta procedure (TP) and Escribano-Jorda procedure (EJP) proposed by [2] and [20], respectively.

2.4.1 Terasvirta procedur

This involves the determination of the lag length (p) of the linear model using Akaike information criterion (AIC), test for the linearity of the conditional mean model and the determination of the delay parameter (d), $d \leq p$ based on the minimum P-value of d. If the null hypothesis of linearity is rejected using the following auxiliary regression proposed by [21] to overcome the problem of unidentified nuisance parameters under the null hypothesis

$$y_t = \beta_0' w_t + \beta_1' w_t y_{t-d} + \beta_2' w_t y_{t-d}^2 + \beta_3' w_t y_{t-d}^3 + \varepsilon_t \tag{15}$$

Then, the choice of transition function is made based on the following sequence of F tests

$H_{02}: \beta_3 = 0$
$H_{03}: \beta_2 = 0|\beta_3 = 0$
$H_{04}: \beta_1 = 0|\beta_2 = \beta_3 = 0$

If H_{02} is rejected implies that the suitable model is LSTAR and that ETAR family of model is rejected. If H_{03} is rejected, it is evidence that the true model is ESTAR model. If the true model is a LSTAR model, then H_{04} is rejected.

2.4.2 Test for ARCH/GARCH Effects

[20] suggest the application of Taylor series approximation of order two to capture the two inflexion points of the exponential function, yielding the auxiliary regression.

$$y_t = \beta_0' w_t + \beta_1' w_t s_t + \beta_2' w_t y_{t-d}^2 + \beta_3' w_t y_{t-d}^3 + \beta_4' w_t y_{t-d}^4 + \varepsilon_t \tag{16}$$

EJP is based on the following two hypotheses within the auxiliary regression (16):
$H_{0L}: \beta_2 = \beta_4 = 0$ with an F-test (F_L)
$H_{0L}: \beta_1 = \beta_3 = 0$ with an F-test (F_E)

If the minimum p-value corresponds to F_E, select LSTAR model, otherwise select ESTAR model. The A**PLSTAR-GARCH** model is the combination of (2), (8), (10) and (12).

STAR models are estimated by nonlinear least squares method, while the estimation of STAR-GARCH models requires two-stage estimation procedure. The two-stage procedure involves the estimation of conditional mean (STAR) model first, followed by the estimation of conditional variance (GARCH) model using residuals from the estimated STAR models with appropriate distribution of the residuals by method of maximum likelihood or quasi maximum estimation if the residuals are Gaussian or non-Gaussian, respectively [22].

2.4.3 Evaluation of Models

Standard error of $\hat{v}_t$ and the value of AIC of different models will be used for comparison of models. Relative forecast performance will be considered as a model selection criterion or an alternative or complement to an in-sample comparison of different models [14]. This is achieved by comparing the forecasts root mean square error (FRMSE) from nonlinear models with that of benchmark linear model. FRMSE is the square root of the Mean square error (MSE) given by

$$\text{FRMSE}=\sqrt{\frac{1}{h}\sum_{i=1}^{h} e_t^2}. \quad (17)$$

3. Hasil dan Diskusi

3.1 Preliminary Analysis

Table 1 shows that exchange rate of naira to Euro is positively skewed and has Platykurtic distribution.

Table 1. Summary of descriptive statistics of exchange rate of naira to euro

Descriptive Statistics	Mean	Std	Min	Max	Median	CV	Skewness	Kurtosis
Naira to Euros	250.50	85.56	148.53	463.84	211.70	0.34	0.93	-0.44

The appearance of time series plot and the slow decay of sample autocorrelation function shown in Figures I and 2, respectively indicate the presence of trend in the series which is an indication of nonstationarity of the series. The Augmented Dickey fuller test (ADF) was applied to test for the presence of unit root in the series. Based on Table 2, there is unit root in the original series since $0.7542 > 0.05$ and the series is stationary by differencing the logarithmic series since $0.01 < 0.05$.

Table 2. Results of augmented dickey-fuller tests

	Original series (Level)			First difference of logarithmic series	
Variable	**Dickey Fuller Statistic**	**ρ- Value**	**Lag Order**	**Dickey Fuller Statistic**	**ρ- Value**
Naira to Euros	-1.5742	0.7542	5	-5.4811	<0.01

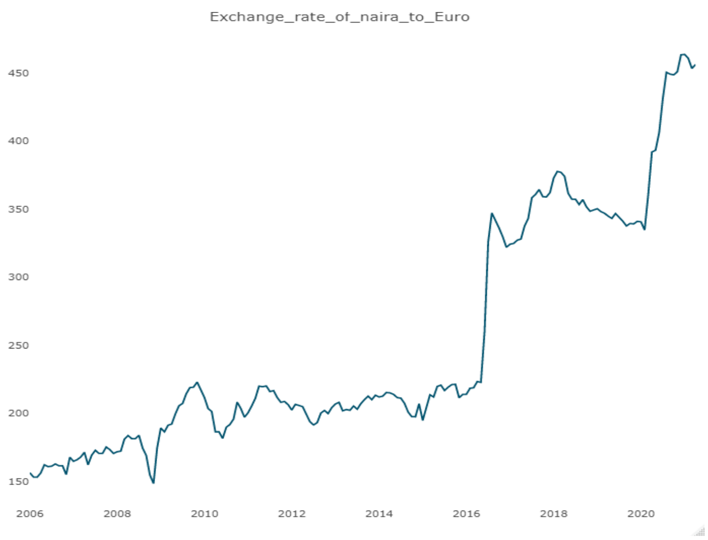

Figure 1. Time series plot of exchange rate of naira/Euro

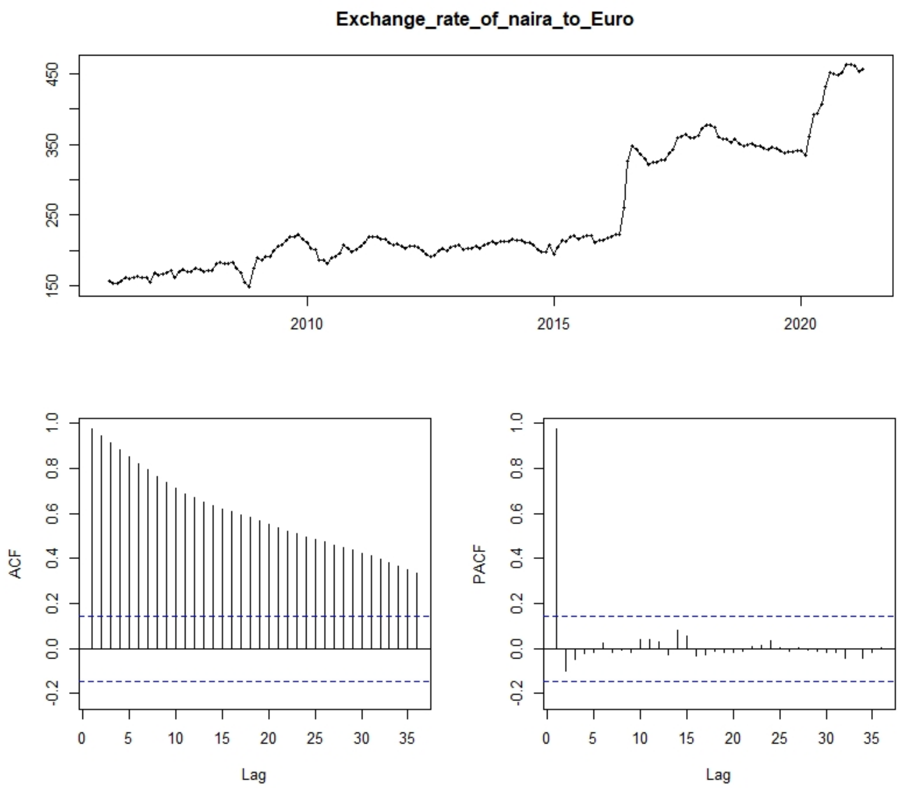

Figure 2. Time series plot, autocorrelation and partial autocorrelation functions of naira/Euro

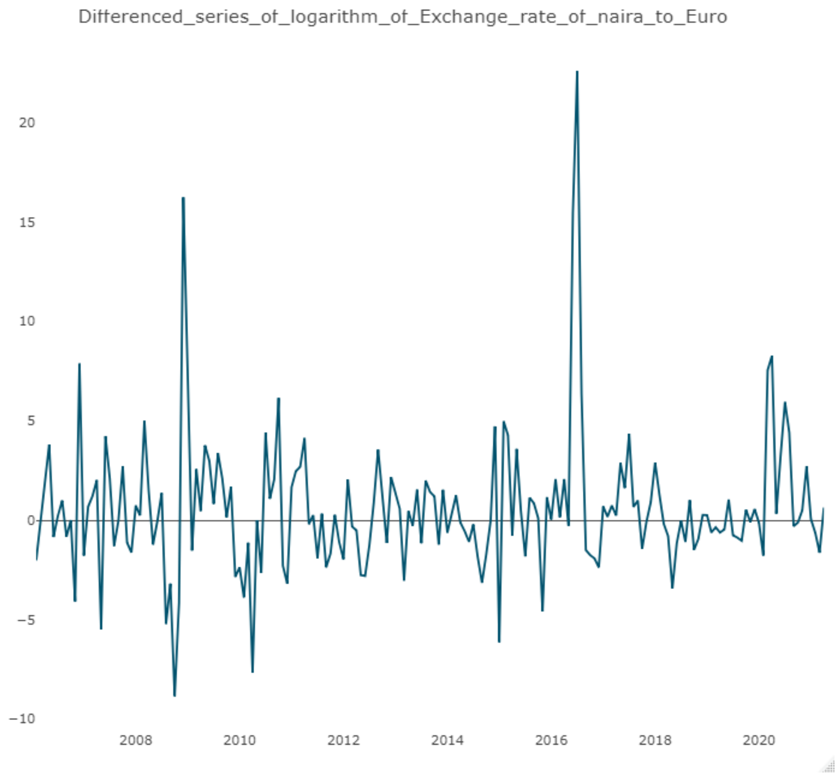

Figure 1. Time series plot of naira/Euro exchange rate percentage return

3.2 Determination of Lag Length of an Autoregressive Model

Table 3 shows that lag 1 has the minimum value of AIC. Hence, ARIMA (1, 1, 0) model is specified for modeling naira/Euro exchange rate percentage return.

Table 3. Lag length of autoregressive model

Lags	Loglik	p(LR)	AIC	BIC	HQC
1	561.38245		-5.621934*	-5.588836*	-5.608538*
2	562.17815	0.20713	-5.619881	-5.570233	-5.599787
3	562.19701	0.84602	-5.610020	-5.543823	-5.583229
4	562.20374	0.90763	-5.600038	-5.517291	-5.566548
5	562.41251	0.51816	-5.592086	-5.492790	-5.551898
6	562.47719	0.71910	-5.582685	-5.466840	-5.535800
7	563.87717	0.09427	-5.586705	-5.454311	-5.533122
8	563.95552	0.69221	-5.577442	-5.428499	-5.517161

In Table 3, the asterisks above indicate the best (that is, minimized) values of Akaike criterion (AIC), Schwarz Bayesian criterion (BIC) and Hannan-Quinn criterion (HQC). Based on Table 4, the estimated residuals from ARIMA (1, 1, 0) model are uncorrelated since the p-value of Box-Pierce statistic $= 0.2248 > 0.05$.

Table 4. Parameter estimates of ARIMA (1, 1, 0) model fitted to exchange rate of naira to Euro percentage return.

Exchange Rate	Model	Z Statistic	Estimate	Box-Pierce Statistic	σ_v	FRMSE
Naira to Euro	ARIMA (1,1,0)	-4.1176	-0.291412 (0.0257996)	1.4734 (0.2248)	3.94815	3.937372

The values in the parentheses are p-values of estimated parameters

3.3 Linearity Test

We test the null hypothesis of linearity against alternative STAR-type nonlinearity using LM test proposed by [2] and the null hypothesis of linearity $H_{01}: \beta_1 = \beta_2 = \beta_3 = \beta_4 = 0$ based on (15) is rejected since $0.0001 < 0.05$ shown in Table 5. Consequently, exchange rate of naira to Euro is specified as nonlinear METS which is modeled with STAR models. The delay parameter is determined to be 1, $d \leq 1$.

Table 5. Result of linearity test

Null hypothesis	F- Statistic	ρ- Value	RSS of y_{t-1}	Regimes
H_{01}	5.614313	0.0001	0.035277	2

Where RSS is the residual sum of squares

3.4 Selection of Transition Function

In accordance with Table 6, $\rho(H_{02}) = 0.0003 < 0.05$ and H_{02} is rejected which implies ESTAR family is rejected. Also, $\rho(H_{03}) = 0.1651 > 0.05$, and H_{02} is not rejected confirming the suitability of LSTAR model with LSTR for modeling exchange rate of naira to Euro. For EJP, $\rho(H_{0E}) = 0.0166 < \rho(H_{0L}) = 0.3955$ suggesting LSTAR model with LSTR1. Both TP and EJP suggest LSTAR model with LSTR1 which is asymmetric STAR model for modeling exchange rate of naira to Euro. Consequently, we fit asymmetric STAR models to exchange of naira to Euro and determine the best model at the evaluation stage using model evaluation measures.

Table 6. Selection of transition function for exchange rate of naira to euro

TP			EJP		
Null Hypothesis	F- Statistic	Transition Function	Null Hypothesis	F- Statistic	Transition Function
H_{02}	13.59052 (0.0003)	First-order logistic function	H_{0L}	0.725466 (0.3955)	First-order logistic function
H_{03}	1.942795 (0.1651)		H_{0E}	4.195249 (0.0166)	
H_{04}	5.443207 (0.0051)				

In Table 6, the values in the parentheses are p values of estimated parameters. $\rho(H_{02})$, $\rho(H_{03})$ and $\rho(H_{04})$ are the p-values of the test correspond to H_{02}, H_{03} and H_{04}, respectively. $\rho(H_{0L})$ and $\rho(H_{0E})$ are the p-values of the test correspond to H_{0L} and H_{0E}, respectively.

3.5 Estimation of STAR Models and Test for the Presence of ARCH Effects in the Residuals of the Estimated STAR Models Fitted to Exchange Rate of Naira to Euro

Table 7 presents the parameter estimates of three asymmetric STAR (ELSTR, APLSTAR and LSTAR) models. In accordance with Table 7, Portmanteau tests reveal that the residuals of the estimated STAR are uncorrelated since the p-values of Box-Pierce Statistic$= 0.2205 > 0.05, 0.5401 > 0.05$ and $0.6156 > 0.05$ for ELSTR, APLSTAR and LSTAR models, respectively. APLSTAR model has the lowest residual standard error and FRMSE, followed by LSTAR model, while ELSTR model has the highest residual standard error and FRMSE. ARCH effects are present using LM tests since the p-value for the three estimated asymmetric STAR models $= 2.2e^{-16} < 0.05$.

Table 7. Parameter estimates of asymmetric STAR models

Parameter	APLSTAR	LSTAR	ESTR
Φ_{10}	0.02231	0.264923	-0.101683
	(0.787)	(0.4742080)	(0.18443)
Φ_{11}	0.42158	0.611009	-0.209619
	(0.026)	(0.00003936)	(0.02438)
Φ_{20}	2.358910	0.463292	-0.153514
	(0.02943)	(0.3409762)	(0.12260)
Φ_{21}	1.20030	-0.590705	-0.611636
	(0.001304)	(0.00889409)	(0.00248)
λ	9.37134	-0.152317	-
	(0.002650)	(0.0001563)	
δ	141.4201	135.069476	0.116942
	(0.6465)	(0.5305777)	(0.0007)
Residuals standard error	3.087	3.1492	3.636
Box-Pierce statistic	0.37533	0.25205	5.242
	(0.5401)	(0.6156)	(0.2205)
FRMSE	3.016371	3.166577	3.553362
Langrage Multiplier Test	4750	4281.7	1453.7
	(< 2.2e-16)	(< 2.2e-16)	(< 2.2e-16)

The values in the parentheses are p-values of estimated parameters.

3.6 Estimation of STAR-GARCH models and test for the remaining ARCH effects in the innovations of the estimated STAR-GARCH models fitted to exchange rate of naira to Euro

Suitable order for STAR-GARCH models are specified for modeling exchange rate of naira to Euro based on AIC, score test and Portmanteau lack of fit test with either Gaussian (normal) or non-Gaussian (std and sstd, ged and sged, etc.) innovations for parameter estimation of STAR-GARCH models. The appropriate distribution of innovations is determined using adjusted Pearson goodness of fit test. The adjusted Pearson goodness-of-fit test is used to compare the empirical distribution of the standardized residuals with the selected theoretical distribution. The null hypothesis is that the empirical and theoretical distribution is identical.

ELSTR-GARCH (1, 1) model is identified for modeling exchange rate of naira to Euro. In Table 8, the parameter estimates of LSTAR-GARCH (1, 1) model are significantly different from zero at 5% level of significance with normal and sged distributions of the innovations. Score test shows no evidence of remaining ARCH effects, while Portmanteau tests of the standardized residuals and standardized squared residuals show no lack of fit for ELSTR -GARCH (1, 1) model. ELSTR -GARCH (1, 1) model with sged innovations fitted well to exchange rate of naira to Euro based on adjusted Pearson goodness of fit test and AIC value.

Table 8. Estimates of the parameters of ELSTR-GARCH (1, 1) model and test for the remaining ARCH effects.

Distribution of residuals	**Parameter**	**Estimate**	**LM Test (ARCH(3))**	**Standardized Residual** $\tilde{v}_t$	**Standardized Residual** $(\tilde{v}_t)^2$	**Adjusted Pearson Goodness of fit test**	**AIC**
Norm	λ_0	3.34499 (0.02625)	0.2180 (0.6406)	7.999 (0.0047)	0.04776 (0.8270)	40.43 (0.0029)	5.284
	λ_1	0.29752 (0.01614)					
	β_1	0.47354 (0.00268)					
Sged	λ_0	1.48513 (0.00000)	0.3841 (0.5354)	8.161 (0.4279)	0.1207 (0.7283)	9.865 (0.956)	5.083
	λ_1	0.28384 (0.00000)					
	β_1	0.63918 (0.000000)					

The values in the parentheses are p-values of estimated parameters.

Based on Table 9, LSTAR-GARCH (1, 1) model is identified for modeling exchange rate of naira to Euro. Significant estimates of the parameters of LSTAR-GARCH (1, 1) model fitted to exchange rate of naira to Euro with normal and ged innovations are obtained. The tests for no remaining ARCH effects in the residuals of the fitted LSTAR-GARCH (1, 1) model using score test is carried out. The results reveal that the residuals of the fitted model is free from ARCH effects. Portmanteau tests of the standardized residuals and standardized squared residuals reveal that the residuals are free from serial correlation. LSTAR-GARCH (1, 1) model with ged innovations fits well to exchange rate of naira to Euro based on adjusted Pearson goodness of fit test and AIC value

Table 9. Estimates of the parameters of LSTAR-GARCH (1, 1) model and test for the remaining ARCH effects.

Distribution of residuals	**Parameter**	**Estimate**	**LM Test (ARCH(3))**	**Standardized Residual**		**Adjusted Pearson Goodness of fit test**	**AIC**
				$\tilde{\nu}_t$	$(\tilde{\nu}_t)^2$		
Norm	λ_0	3.06523 (0.001910)	0.3693 (0.5434)	0.08218 (0.7744)	0.00802 (0.9286)	47.18 (0.00034)	5.0643
	λ_1	0.297519 (0.016138)					
	β_1	0.473539 (0.002683)					
Ged	λ_0	0.025956 (0.04332)	0.1741 (0.6765)	0.2892 (0.5907)	6.041 (0.1398)	38.94 (0.10273)	4.9350
	λ_1	0.29411 (0.019601)					
	β_1	0.45483 (0.000432)					

The values in the parentheses are p-values of estimated parameters.

Table 10. Parameter estimates of APLSTAR-GARCH (0, 1) model and test for the remaining ARCH effects.

Type of Distribution	**Parameter**	**Estimate**	**LM Test (ARCH(2))**	**Standardized Residual**		**Adjusted Pearson Goodness of fit test**	**AIC**
				$\tilde{\nu}_t$	$(\tilde{\nu}_t)^2$		
Normal	λ_0	0.00000 (0.99998)	1.733 (0.1880)	0.5688 (0.4507)	0.1241 (0.7246)	49.87 (0.0167)	5.0552
	β_1	0.999000 (0.00000)					
Student's t-distribution	λ_0	0.00003 (0.000218)	0.7092 (0.3997)	0.9141 (0.3390)	0.1258 (0.7228)	12.11 (0.8808)	4.7467
	β_1	0.99663 (0.000000)					

The values in the parentheses are p-values of estimated parameters.

In accordance with Table 10, APLSTAR-GARCH (0, 1) model is identified for modeling exchange rate of naira to Euro with normal and std innovations. The residuals obtained from fitting APLSTAR-GARCH (0, 1) model to exchange rate of naira to Euro show no remaining ARCH effects using score tests. Also, the standardized residuals and standardized squared residuals are uncorrelated using Portmanteau tests. APLSTAR-GARCH (0, 1) model with std innovations fits well to exchange rate of naira to Euro. Based on Table 11, the FRMSE indicates that the three asymmetric STAR-GARCH models fitted to exchange rate of naira to Euro outperform the linear model. APLSTAR-GARCH (1, 1) model has the smallest FRMSE and AIC value, followed by LSTAR-GARCH (1,1) model.

In accordance with Tables 7 and 11, the FRMSE of STAR models are higher than the FRMSE of their corresponding STAR-GARCH models. Consequently, STAR-GARCH model outperform its constituent models in modeling METS.

Table 11. Evaluation of forecasts generated by AR and STAR-GARCH models fitted to exchange rate of naira to Euro.

Estimated models	**FRMSE**	**FRMSE Ratio=** $\frac{\mathbf{FRMSE_{NL}}}{\boldsymbol{F}\mathbf{RMSE_{L}}}$	**AIC**
ELSTR-GARCH(1,1)	3.453387	0.877079	5.0833
APLSTAR-GARCH(0,1)	3.00569	0.763375	4.7467
LSTAR-GARCH(1,1)	3.166577	0.804236	4.9350
ARIMA(1,1,0)	3.937372	-	

Where $\mathrm{FRMSE_L}$ and $F\mathrm{RMSE_{NL}}$ are the forecast root mean square errors of linear and nonlinear models, respectively.

3.7 Discussion of Findings

Monthly exchange rate of naira to Euro extracted from the Central Bank of Nigeria (CBN) statistical bulletin spanning from January, 2006 to April, 2021 are used for empirical illustrations after transformation. Time series plots of exchange rate of naira to Euro indicates eminent of volatility clustering from 2016 to 2020 which could be due to Federal Government policies, economic recession and Covid-19 pandemic.

The specification of conditional mean model was based on TP and EJP. Both TP and EJP suggest first-order logistic function (asymmetric) for modeling exchange rate of naira to Euro. Hence, three asymmetric STAR (LSTAR, ELSTR and APLSTAR) models are fitted to exchange rate of naira to Euro. The Portmanteau tests reveal that the residuals from these models are free from serial correlation indicating the adequacy of these models. APLSTAR model has the lowest residual standard error and RMSE, followed by LSTAR model, while ELSTR model has the highest residual standard error and FRMSE. The implication of these results is that asymmetric models (APLSTAR, ELSTR and LSTAR) can account for the nonlinear features of exchange rate which is known to exhibit symmetric behaviour [11, 12]. However, [23] who modeled real exchange rate to determine the validity of purchasing power parity using TAR and ESTAR (symmetric) models suggested that other nonlinear models should be considered for modeling exchange rates.

The presence of ARCH effects in the residuals of these three STAR models fitted to exchange rate of naira to Euro is ascertained using LM tests. Appropriate STAR-GARCH models are specified for modeling exchange rate of naira to Euro. LSTAR-GARCH (1,1), ELSTR-GARCH(1,1) and APLSTAR-GARCH (0,1) models with ged, sged and std innovations, respectively are identified and estimated based on AIC and adjusted Pearson goodness of fit tests. There was no evidence of remaining ARCH effects based on score tests. Portmanteau tests of null hypothesis of no serial correlation in the standardized residual and standardized squared residuals reveal no lack of fit of these STAR-GARCH models. RMSE obtained indicates that the three asymmetric STAR-GARCH models fitted to exchange rate of naira to Euro outperformed the linear model and APLSTAR-GARCH (0,1) model is the best, followed by LSTAR-GARCH(1,1) model.

Conclusion

Asymmetric power logistic smooth transition autoregressive (APLSTAR) model characterize exchange rate of naira to Euro known to be symmetric. Consequently, the efficiency of STAR models varies from series to series and depend largely on the transition function; the hybrid STAR-GARCH model is more efficient than the constituent model; APLSTAR-GARCH (0, 1) model is the best for modeling exchange rate of naira to Euro.

References

[1] Effiong, B A., Okereke, E.W., Omekara, C. O., Acha, C. K., & Akpan, E. A. A New Family of Smooth Transition Autoregressive (STAR) Models: Properties and Application of its Symmetric Version to Exchange Rates. *Communication in Physical Sciences*, 9(3), 310-324, 2023.

[2] Terasvirta, T. Specification, Estimation & Evaluation of STAR Models. *Journal of the American Statistical Association*, 89(425), 208- 218, 1994.

[3] Box, E. P., Jenkins, G. M., Reinsel, G. C., & Ljung, G. M. *Time Series Analysis Forecasting and Control*. John Wiley and Sons, Inc. Hoboken, Jersey. 361-370, 2016.

[4] Hamilton, J. D. *Time Series Analysis*. Princeton University Press, Princeton, 1994.

[5] Tong, H. & Lim, K. S. Threshold autoregressive, Limit Cycles and Cyclical Data. *Journal of Royal Statistics Society*, 42(3), 245-292, 1980.

[6] Taylor, M. P. & Sarno, L. The Behaviour of Real Exchange Rates during the Post-BrettonWood Period. *Journal of International Economies*, 46, 261-312, 1998.

[7] Sarantis, N. Modeling Nonlinearities in Real Effective Exchange Rates. *Journal of International Money and Finance*, 18, 27-45, 1999.

[8] Taylor, M. P. & Peel, D. A. Nonlinear Adjustment, Long-Run Equilibrium and Exchange Rate Fundamentals. *Journal of International Money and Finance*, 19, 33-53, 2000.

[9] Sarno, l. *Real Exchange Rate Behaviour in the Middle East: A ReExamination*. Applied Economics Letters, 66, 127-13, 2000.

[10] Baum, C. F., Barkoulas, J. T., & Caglayan, M. Nonlinear Adjustment to purchasing Power Parity in the Post-Bretta Woods Era. *Journal of International Money and Finance*, 20, 379-399, 2001.

[11] Liew, V. K-S., Baharumslah, A. Z., & Lau, S-H. Forecasting Performance of Logistic STAR Exchange Rate Model: The Original and Reparameterized Versions. *MPRAS Paper*, No 511, 2003.

[12] Baharumshah, A. Z. & Liew, V. K-S. Forecasting Performance of Exponential Smooth Transition Autoregressive Exchange Rate. *Open Economies Review*, 17, 235-25, 2006.

[13] Oyewale, A. M. On Evaluating the Volatility of Nigerian Gross Domestic Product Using Smooth Transition Autoregressive-GARCH (STAR - GARCH) Models. *American Journal of Applied Mathematics and Statistics*, 9(3), 102-106, 2021.

[14] Dijk, D. V., Terasvirta, T. & Franses, P. H. Smooth Transition Autoregressive models – A survey of Recent Developments. *Econometric Reviews*, 21(1): 1- 47, 2002.

[15] Jansen, E. S. & Terasvirta, T. Testing Parameter Constancy and Super Exogeity in Econometric Equations. *Oxford Bulletin of Economies and Statistics*, 58:735-763, 1996.

[16] Shangodoyin, D.K., Adebille, O. A., & Arnab, R. Specification of Logistic STAR Model: An Alternative Representation of the Transition Function. *Journal of Mathematics and Statistics*, 54(4): 251-259, 2009.

[17] Engle, R. F. Autoregressive conditional heteroscedasticity with estimates of the Variance of United Kingdom inflation, *Econometrica*: *Journal of the econometric society*, 50, 987-1008, 1982.

[18] Tsay, R. S. *Analysis of Financial Time Series*. John Wiley and Sons, Inc. 605 Third Avenue, New York, 83-87, 2002.

[19] Bollerslev, T. Generalized autoregressive conditional heteroskedasticity, *Journal of Econometrics*, 31, 307–327, 1986.

[20] Escribano, A. & Jorda, O. Testing nonlinearity. Decision Rules for Selecting between Logistic and Exponential STAR Models. *Spanish Economic Review*, 3: 193-209, 2001.

[21] Luukkonen, R., Saikkonen, P., & Terasvirta, T. Testing Linearity against STAR Models. *Biometrika*, 75(3), 491- 499, 1988.

[22] Chan, F. & McAleer, M. Estimating Smooth Transition autoregressive models with GARCH errors in the presence of Extreme Observations and Outliers. *Applied Financial Economics*, 13(8), 581- 592, 2003.

[23] Yoon, G. Do Real Exchange Rates Really Follow Threshold Autoregressive Model or Exponential Smooth Transition Autoregressive Models? *Economic Modeling,* 27, 605-612, 2010.