Comparison of Variance Covariance and Historical Simulation Methods to Calculate Value At Risk on Banking Stock Portfolio

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Abstract
In investing, all investors must be faced with risk that must be borne. Therefore, to determine the best strategy in investing, every investor must calculate the risk. One statistical approach that can be used to measure the risk is Value at Risk (VaR). VaR is defined as a tolerable loss with a certain level of confidence. The purpose of this research is to estimate VaR using Variance Covariance and Historical Simulation methods on banking stock portfolio consisting of three stocks for the period 11 September 2020-30 September 2021. Both methods will then be evaluated using backtesting to determine the accuracy of VaR and to obtain the best method. From the research results, if the holding period is 1 day, then the VaR calculation for banking stock portfolio using both methods can be used to estimate the risk at 99% and 95% confidence levels, except for the VaR value using the Variance Covariance method for banking stock portfolio at 95% confidence level. The results show that Variance Covariance method is the best method for 99% confidence level. As for the 95% confidence level, Historical Simulation method is the best method.

Keywords: Value at Risk, Variance Covariance, Historical Simulation, Backtesting, Banking Stock Portfolio.

1. INTRODUCTION AND PRELIMINARIES

Investment in stock is an allocation of assets owned with the hope of getting profits in the future [1]. The profit obtained is a good side of the investment. However, in investing, investors will not always experience profits. This is because all investments contain an element of uncertainty or risk. According to Devi (2010), risk can be characterized as the greatness of the deviation between the expected and the actual rate of return, so that risk is an uncertainty about the outcomes to be acquired in the future because of in general changes in economic situations. Due to this uncertainty, investors often ask about the amount of risk or loss that will be borne if market conditions turn out to be bad. Therefore, it is necessary to calculate the risk so that investors can find out the value of risk early. One of the risk measurements that is often used in industries is Value at Risk (VaR) [2]. Value at Risk is a measurement of the maximum possible
loss in period $T$ with a certain level of confidence $\alpha$ and the VaR calculation is largely determined by the distribution of returns [3].

To calculate or estimate the VaR, there are two fundamental methods that can be used, namely Variance Covariance method and Historical Simulation method. Variance Covariance method assumes that returns are normally distributed and portfolio returns are linear with respect to single asset returns [4]. While the VaR with Historical Simulation is a method that abrogates the supposition of returns that are normally distributed and linear between portfolio returns and single asset return [5]. After knowing the methods for calculating Value at Risk, investors also often ask about which method should be chosen to produce an accurate VaR value. Therefore, after calculating the VaR, it is necessary to do the validity test using Backtesting. Backtesting is a formal measurable method that comprises of checking the actual losses are in accordance with predicted losses [6]. This method can also be used to find the best method according to the VaR values in certain stocks.

In this study, the Variance Covariance method and the Historical Simulation method will be compared and used to calculate the VaR of banking stock portfolio consists of three stocks, namely PT. Bank Negara Indonesia Tbk, PT. Bank Rakyat Indonesia Tbk, and PT. Bank Central Asia Tbk. These companies are LQ45 companies recorded on the Indonesia Stock Exchange for period August 2020 to February 2022. The stocks that are inputted in LQ45 are stocks that have high liquidity, huge market capitalization and are supported by good fundamentals [7]. In addition, the three banking stocks mentioned above are also the stocks that are most targeted and collected by the investors [8]. For these reasons, it is interesting for investors to know the investment prospects in these companies. The application and the comparison of VaR calculation using Variance Covariance and Historical Simulation methods on portfolio stocks of the financial sector to determine the portfolio risk formed from their single assets.

Investment can be defined as a commitment to put sum of money or other resources made at this time with the hope of obtaining benefits in the future [9]. There are two factors considered in making an investment, namely the expected rate of return and risk [10]. Almost all investments contain an element of uncertainty so that investors do not know the results they will get. One of the most popular investment instruments is stock investment. Stocks are securities that show ownership of one or many companies. Stocks can be characterized as a symbol of participation or ownership of a private or corporate in a company [11].

In general, risk is associated with the possibility of an undesirable adverse outcome or loss occurring [12]. In investing, risk arises due to changes or fluctuations in asset prices so that the investment returns to be received deviate from the expected profits. The relationship between return and risk is that the higher the return, the higher the risk. Therefore, it is necessary for investors to anticipate the level of risk that provides high returns [10].

Return is the profit obtained by companies, individuals, or institutions from the results of investment policies made [9]. According to Jorion (2007), return can be calculated as follows:

$$ R_t = \frac{P_t - P_{t-1}}{P_{t-1}} $$  \hspace{1cm} (1.1)

where, $R_t$ is return in period $t$, $P_{t-1}$ is asset price in period $t - 1$, and $P_t$ is asset price in period $t$.

According to Syariah and Pratiwi (2020), stock volatility is the standard deviation value of the return to measure the level of risk of a stock. The level of risk is represented by the size of the deviation or the size of the spread of the data. Two measures of spread that are often used to represent it are the value of the variance and the standard deviation. The calculation of variance and standard deviation can be done using the following formula:
where, $\sigma^2$ is variance of return, $\sigma$ is standard deviation, $R_t$ is return at time or period $t$, $E(R)$ is the mean or average return, and $T$ is the total or number of periods during the transaction.

In investing, investors do not invest only in one asset, but in various types of assets. This collection of assets or stocks is known as a portfolio. By compiling a portfolio, investors hope that the level of income obtained will be optimal with the level of risk that is suppressed to a minimum [1]. The return of the portfolio consisting of various kinds of assets can be calculated by the following formula [13]:

$$R_{p,t} = \sum_{i=1}^{N} w_i R_{i,t}, t = 1,2, \ldots, T$$  

where, $R_{p,t}$ is return of portfolio in period $t$, $w_i$ is weight or proportion of asset $i$, $R_{i,t}$ is return of asset $i$ in period $t$, $N$ is number of single assets in portfolio, and $T$ is the total or number of periods during the transaction.

The weighting vector is used so that the portfolio has a minimum variance, meaning that the expected return of the constituent assets does not differ much from the overall portfolio that can be formed so that it can produce the smallest VaR value from the existing possibilities. The formula of the portfolio weight can be calculated as follows [10]:

$$w = \frac{s^{-1}1_N}{1_N^T S^{-1} 1_N} = \begin{bmatrix} w_1 \\ \vdots \\ w_N \end{bmatrix}$$

where:

- $w$ = the weight
- $S^{-1}$ = the inverse of the covariance matrix of $N \times N$
- $1_N = a matrix NX1 with element 1 as much as N$
- $N$ = the total of assets used in the portfolio

**Variance Covariance Method**

Variance Covariance method is a method that assumes that the percentage change in asset prices has a normal distribution. Before calculating the VaR, it is necessary to calculate the covariance matrix, variance, and standard deviation [14]. The formula to find the covariance matrix can be written as follows:

$$S = COV(X) = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \cdots & \sigma_{1N} \\ \sigma_{21} & \sigma_{22} & \cdots & \sigma_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{N1} & \sigma_{N2} & \cdots & \sigma_{NN} \end{bmatrix}$$

where, $\sigma_{ij}$ represents the covariance of asset $i$ and $j$, $N$ is number of single assets in portfolio

While the formulas to calculate the variance and standard deviation of the portfolio consisting of $N$ assets are as follows:

$$\sigma^2_p = [w_1 \ w_2 \ \cdots \ w_N] \begin{bmatrix} \sigma_{11} & \sigma_{12} & \cdots & \sigma_{1N} \\ \sigma_{21} & \sigma_{22} & \cdots & \sigma_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{N1} & \sigma_{N2} & \cdots & \sigma_{NN} \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_N \end{bmatrix} = w^T S w$$
After the covariance matrix, variance, and standard deviation are obtained, the VaR of the portfolio with Variance Covariance method can be estimated. The formula to estimate the VaR of the portfolio with Variance Covariance method is as follows [15]:

\[ \text{VaR} = Z_{(1-\alpha)} \sigma_p \sqrt{hp} \]

where, \( P \) is initial investment, \( Z_{(1-\alpha)} \) = standard normal distribution quantile with confidence level \( (1 - \alpha) \), \( hp \) = holding period, and \( \sigma_p \) = the standard deviation of portfolio

**Historical Simulation Method**

Historical Simulation method is a method that can be directly used to calculate the VaR because this method does not require assumption about normality. According to Dowd (2005), the formula that can be used in calculating the VaR is as follows:

\[ \text{VaR} = P \times \text{Percentile} \ 100 \alpha \times \sqrt{hp} \] (1.10)

with significance level of \( \alpha \).

After VaR is obtained, the validity or the accuracy of VaR is tested using backtesting method as explained in [16]. According to Jorion (2007), the VaR value is only useful if it can predict the risk well.

**2. MAIN RESULTS**

The population in this study were taken from companies listed on the Indonesia Stock Exchange and contained in LQ45 stocks for period August 2020 to February 2022. This research is utilizing purposive sampling technique to obtain the data. The data used in this study is the daily stock closing prices for 256 days on PT. Bank Negara Indonesia Tbk (BBNI), PT. Bank Rakyat Indonesia Tbk (BBRI), and PT. Bank Central Asia Tbk (BCA) period September 11, 2020 to September 30, 2021. In addition, the source of data used in this study is obtained from [www.yahoofinance.com](http://www.yahoofinance.com).

This research calculated the VaR using Variance Covariance and Historical Simulation methods. The software used for data analysis are R Studio version 4.0.2 and Microsoft Excel 2016. The steps or research procedures carried out are as follows:

1. Collect the data needed in this research.
2. Finding the return values of each stock by using equation 1.1, these data will later be used for the calculation of the portfolio returns.
3. Determine the confidence level.
4. Determine the holding period.
5. Finding the covariance matrix of each single asset return with equation 1.6, it is needed in determining the VaR using the Variance Covariance method. After the covariance matrix is built, the inverse of the covariance matrix can be found to calculate the weight for each stock in the portfolio.
6. Finding the weight for each asset in the portfolio to get the maximum portfolio using the equation 1.5
7. Calculate the portfolio returns which are consisting of the returns on each single asset with the weights calculated in the previous step. The calculation of portfolio returns can be done by using the equation 1.4. After portfolio returns have been obtained, they can be sorted from the lowest to the highest.
8. Calculate the variance and standard deviation of the portfolio by using equation 1.7 and 1.8. These are needed in estimating the Value at Risk with Variance Covariance method.
9. Perform normality test of portfolio stock returns data using Lilliefors test. If the data are normal, the next step can be done. However, if the data are not normal, the Cornish-Fisher Expansion approach can be used. The Cornish-Fisher expansion formula is
   \[ Z^* = Z + \frac{1}{6} (Z^2 - 1)S_k + \frac{1}{24} (Z^3 - 3Z)(K_r - 3) - \frac{1}{36} (2Z^3 - 5Z)S_k^2 \]
   where, \( Z^* \) is the new standard normal distribution quantile, \( Z \) is the standard normal distribution quantile, \( S_k \) is the skewness, and \( K_r \) is the kurtosis.
10. Determining the VaR values of a portfolio with Variance Covariance and Historical Simulation methods by using equation 1.9 and 1.10.
11. Doing the validity test with backtesting.
12. Selecting the best method by looking at the accuracy that was obtained from backtesting. If all methods are accurate, then the best method can be seen from the minimum number of exceptions and the largest VaR [16].
13. Make interpretations and conclusions from the results that have been obtained from the VaR calculation and backtesting.

Stock Returns Calculation
After the daily stock closing prices of 3 baking stocks have been collected, the daily stock returns can be calculated using equation 1.1. To determine the characteristic and the pattern of the data, descriptive statistics can be used. The following are the results of descriptive statistics for 3 banking stocks for the period 11 September 2020 until 30 September 2021:

<table>
<thead>
<tr>
<th>Code</th>
<th>Mean</th>
<th>Minimum</th>
<th>Maximum</th>
<th>Standard Deviation</th>
<th>Sum Return</th>
<th>Obs.</th>
</tr>
</thead>
<tbody>
<tr>
<td>BBNI</td>
<td>0.0006102</td>
<td>-</td>
<td>0.0677966</td>
<td>0.02123478</td>
<td>0.1556038</td>
<td>255</td>
</tr>
<tr>
<td></td>
<td>0.0590476</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>BBRI</td>
<td>0.0009112</td>
<td>-</td>
<td>0.0840108</td>
<td>0.02228195</td>
<td>0.2323483</td>
<td>255</td>
</tr>
<tr>
<td></td>
<td>0.0909248</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>BBCA</td>
<td>0.0007836</td>
<td>-</td>
<td>0.0638298</td>
<td>0.01533876</td>
<td>0.1998068</td>
<td>255</td>
</tr>
<tr>
<td></td>
<td>0.0400000</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Confidence Level and Holding Period Estimation
In this research, the levels of confidence used to calculate the VaR for a portfolio consisting of 3 banking stocks (BBNI, BBRI and BBCA) are 99% and 95%. While for the holding period used in this research is 1 day. This 1 day holding period is chosen because the longer the investor holds an asset, the VaR calculation will be more inaccurate.
Portfolio Weight or Proportion

Before calculating the portfolio weight or proportion, the covariance matrix must be searched first by using equation 1.6. With the help of R Studio software, the result of the covariance matrix for banking stocks can be seen as follows:

$$
S = \begin{bmatrix}
BBNI & BBRI & BBCA \\
BBNI & 0.0004509157 & 0.0002953429 & 0.0001749962 \\
BBRI & 0.0002953429 & 0.0004964854 & 0.0001623852 \\
BBCA & 0.0001749962 & 0.0001623852 & 0.0002352777 \\
\end{bmatrix}
$$

After getting the covariance matrix for banking stocks, then looking for the inverse of the covariance matrix using the R Studio software. The inverse of the covariance matrix for banking stocks can be seen as follows:

$$
S^{-1} = \begin{bmatrix}
BBNI & BBRI & BBCA \\
BBNI & 4141.033 & -1880.472 & -1782.1682 \\
BBRI & -1880.472 & 3455.3279 & -986.1473 \\
BBCA & -1782.1682 & -986.1473 & 6256.4730 \\
\end{bmatrix}
$$

Since the inverse of the covariance matrix for banking stocks have been obtained, the weight or proportion of funds for each asset can be found using equation 1.5. Because in this research, there are 3 stocks, then the amount of \( N \) used in equation 1.3 is 3 so that \( 1_N = 1_3 \).

$$
1_3 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}
$$

$$
1_3^T = [1 \ 1 \ 1]
$$

The weight or proportion of each asset for banking stock portfolio can be seen as follows:

$$
w = \begin{bmatrix} 0.1050199 \\ 0.1292371 \\ 0.7657430 \end{bmatrix}
$$

From the results obtained, the weight or proportion for the banking stocks of BBNI is 0.1050199 or 10.502%, BBRI is 0.1292371 or 12.924%, BBCA is 0.7657430 or 76.574%.

Portfolio Stock Returns Calculation

After getting the weight or proportion of funds for each asset, then the returns for banking stock portfolio can be calculated using equation 4. To see the stock movement of the portfolio, a time series plot can be used which is one of the simplest visual statements. The time series plot of banking stock portfolio can be seen as follows:

Figure 2.1. The Time Series Plot of Banking Stock Portfolio Returns
From Figure 2.1, it can be seen that the stock returns data movement of the banking stock portfolio is very volatile, where from the beginning to the end of the period, the stock returns movement is between -0.03 and 0.06 with an average return price of 0.00078 and the total return price is 0.1994. The plot of the returns of banking stock portfolio above shows that the return on portfolio stock varies widely, where there is a very high return and a very low return. After calculating the returns on the banking stock portfolio, the returns can be sorted from the smallest to the largest. The returns need to be sorted for the calculation of Value at Risk using Historical Simulation method.

**Portfolio Stock Volatility Calculation**

To get the volatility or standard deviation, the variance must be calculated first. The calculation of variance for banking stock portfolio is by using equation 1.7. After getting the variance value, the standard deviation can be calculated by using equation 1.8. The result of the standard deviation will be used later for the calculation of VaR using the Variance Covariance method. The results of the calculation of the variance and standard deviation values can be seen in the following table:

<table>
<thead>
<tr>
<th>Variable</th>
<th>Banking Stock Portfolio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variance</td>
<td>0.0002195265</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>0.01481643</td>
</tr>
</tbody>
</table>

**Portfolio Stock Returns Normality Test**

Since returns are assumed to be normally distributed for the Variance Covariance method, then before calculating the Value at Risk, the data normality assumption test is first performed for the banking stock portfolio by using the Lilliefors test to determine whether the returns of the banking stock portfolio follow a normal distribution. From the calculation result of the Lilliefors test, the returns of the banking stock portfolio obtained a $p-value$ of 0.0003314. Since $p-value < \alpha$ both for $\alpha=5\%$ and $\alpha=1\%$, then $H_0$ is rejected. Because banking stock portfolio returns with the confidence levels of 99% and 95% indicate that the data are not normally distributed, it is necessary to calculate Cornish-Fisher Expansion to obtain the new standard normal distribution quantile so that the Value at Risk calculation using the Variance Covariance method can be done. The calculation of Cornish-Fisher Expansion can be done using the equation 1.11. The results obtained from the calculation of Cornish-Fisher Expansion can be seen as follows:

<table>
<thead>
<tr>
<th>Portfolio Stock Portfolio (CL = 99%)</th>
<th>Z</th>
<th>Skewness</th>
<th>Kurtosis</th>
<th>Cornish-Fisher Expansion</th>
</tr>
</thead>
<tbody>
<tr>
<td>Banking Stock Portfolio (CL = 99%)</td>
<td>2.326348</td>
<td>0.7596136</td>
<td>1.35284</td>
<td>2.282666</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Portfolio Stock Portfolio (CL = 95%)</th>
<th>Z</th>
<th>Skewness</th>
<th>Kurtosis</th>
<th>Cornish-Fisher Expansion</th>
</tr>
</thead>
<tbody>
<tr>
<td>Banking Stock Portfolio (CL = 95%)</td>
<td>1.644854</td>
<td>0.7596136</td>
<td>1.35284</td>
<td>1.883182</td>
</tr>
</tbody>
</table>

**Value at Risk Estimation Using Variance Covariance Method and Historical Method**
After the standard deviation and the new standard normal distribution quantiles are known, Value at Risk using the Variance Covariance method can be estimated by equation 1.9. While for the Historical Simulation method, VaR can be estimated from the sorted of banking stock portfolio returns. The results of the Value at Risk estimation using Variance Covariance and Historical Simulation methods for banking stock portfolio can be seen in the following table:

<table>
<thead>
<tr>
<th>CL</th>
<th>Variance Covariance method</th>
<th>Historical Simulation method</th>
</tr>
</thead>
<tbody>
<tr>
<td>VaR</td>
<td>0.03382096</td>
<td>0.02991714</td>
</tr>
<tr>
<td>P</td>
<td>Rp 100.000.000</td>
<td>Rp 100.000.000</td>
</tr>
<tr>
<td>VaR (Rp)</td>
<td>Rp 3.382.096</td>
<td>Rp 2.991.714</td>
</tr>
</tbody>
</table>

From Table 2.4, for Variance Covariance method, it can be interpreted that with an initial investment of Rp 100.000.000 at 99% confidence level, the risk that investors may suffer will not exceed Rp 3.382.096 within one day after 30 September 2021, or in other words, it can be interpreted that at the 1% significance level, the investment risk in a banking stock portfolio is Rp 3.382.096. Meanwhile, for a 95% confidence level, the risk that may be suffered by investors will not exceed Rp 2.790.203 within one day after 30 September 2021, or in other words, it can be interpreted that at 5% significance level, the investment risk is Rp 2.790.203. While for Historical Simulation method, it can be interpreted that with an initial investment of Rp 100.000.000 for a banking stock portfolio at 99% confidence level, the risk that investors may suffer will not exceed Rp 2.991.714 within one day after 30 September 2021, or in other words, it can be interpreted that at the 1% significance level, the investment risk in the banking stock portfolio is Rp 2.991.714. Meanwhile, for a 95% confidence level, the risk that may be suffered by investors will not exceed Rp 1.977.972 within one day after 30 September 2021, or in other words, it can be interpreted that at 5% significance level, the investment risk in a banking stock portfolio is Rp 1.977.972.

**Best Method Selection**

After estimating Value at Risk, the best method between these two methods can be known by doing the validity test, which is the backtesting. Backtesting can determine whether the Value at Risk result is accurate or not. The number of failures in backtesting is obtained by calculating the actual losses that exceed the VaR calculation result on the stock portfolio returns data. The result of backtesting are as follows:

<table>
<thead>
<tr>
<th>Backtesting</th>
<th>Variance Covariance method</th>
<th>Historical Simulation method</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\alpha = 0.01$</td>
<td>$\alpha = 0.05$</td>
</tr>
<tr>
<td></td>
<td>$\alpha = 0.01$</td>
<td>$\alpha = 0.05$</td>
</tr>
<tr>
<td>Total Observation</td>
<td>255</td>
<td>255</td>
</tr>
<tr>
<td>Number of Failure</td>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>Significance Level</td>
<td>0.01</td>
<td>0.05</td>
</tr>
<tr>
<td>Likelihood Ratio</td>
<td>-</td>
<td>6.3844</td>
</tr>
</tbody>
</table>
Based on the result of backtesting above it can be concluded that for a 99% confidence level, the VaR values using the Variance Covariance and Historical Simulation methods are accurate. Meanwhile, for the 95% confidence level, the VaR value using the Variance Covariance method is not accurate. However, the VaR value is accurate for Historical Simulation method.

3. CONCLUSION

Based on the calculation and testing of the accuracy of VaR, the Variance Covariance method is the best method for a 99% confidence level. As for the 95% confidence level, the Historical Simulation method is the best method. For investors, it is advisable to pay attention to the level of confidence they have and consider adding or changing the shares to be selected in the formation of a portfolio if the Value at Risk obtained in this study is considered too high. As for further research, it is recommended to extend the period of closing stock price data to be investigated and also to use a different method such as the Monte Carlo method, so that the method can be compared with the methods that have been applied in this study. In addition to other methods, future research can also continue to calculate the Expected Shortfall or Tailed Value at Risk (TVaR) as a measure to predict risk beyond the VaR.

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