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Total Rainbow Connection Number Of Shackle Product Of Antiprism Graph (AP_3)

Bilangan Terhubung Total Pelangi Pada Graf Hasil Operasi *Shackle* Graf Antiprisma (AP_3)

Melisa Huntala[1)*], Muhammad Rezky F. Payu[2)], Nisky Imansyah Yahya[3) *]

[1,3)]Program Studi Matematika, Fakultas Matematika dan Ilmu Pengetahuan Alam, Indonesia*
[2)]Program Studi Statistika, Fakultas Matematika dan Ilmu Pengetahuan Alam, Indonesia
[3)]Algebra and Combinatorial Mathematics Research Group, Fakultas Matematika dan Ilmu Pengetahuan Alam
***Email**: [1)*]melisahuntala10@gmail.com, [2)] rezky@ung.ac.id, [3)]nisky@ung.ac.id*

Abstract

Function if $c : G \rightarrow \{1,2,\dots,k\}$ is said to be k total rainbows in G, for each pair of vertex $V(G)$ there is a path called $x - y$ with each edge and each vertex on the path will have a different color. The total connection number is denoted by trc(G), defined as the minimum number of colors needed to make graph G to be total rainbow connected. Total rainbow connection numbers can also be applied to graphs that are the result of operations. The denoted shackle graph $(G_1, G_2, \dots, G_t)$ is a graph resulting from the denoted graph $G\ (G, t)$ where t is number of copies of G. This research discusses rainbow connection numbers rc(G) and total rainbow connection trc(G) using the shackle operation, where G is the antiprism graph(AP_3). Based on this research, rainbow connection numbers rc(shack $AP_3, t) = t + 2$, and total rainbow connection trc(shack$(AP_3, t) = 2t + 3$ for $t \geq 2$.

Keywords: Total Rainbow Connection, Shackle, Antiprism Graph

Abstrak

Fungsi jika $c : G \rightarrow \{1,2,\dots,k\}$ dikatakan k total pelangi pada G, untuk setiap pasang titik $V(G)$ terdapat lintasan disebut x-y dengan setiap sisi dan setiap titik pada lintasan akan memiliki warna berbeda. Bilangan terhubung total pelangi dilambangkan dengan trc(G), didefinisikan sebagai jumlah minimum warna yang diperlukan untuk membuat graf G menjadi terhubung-total pelangi. Bilangan terhubung total pelangi juga dapat diterapkan pada graf yang merupakan hasil operasi. Graf shackle yang dilambangkan $(G_1, G_2, \ldots, G_t)$ adalah graf yang dihasilkan dari graf G yang dilambangkan (G, t) dengan t adalah jumlah salinan dari G. Penelitian ini membahas mengenai bilangan terhubung pelangi rc(G) dan bilangan terhubung total pelangi trc(G) menggunakan operasi shackle, dimana G adalah graf Antiprisma (AP_3) Berdasarkan penelitian ini, diperoleh bilangan terhubung pelangi rc(shack AP_3, t) $=$ $t+2$, dan total pelangi trc(shack $AP_3, t) = 2t +$ 3 untuk $t \geq 2$.

Kata kunci: Bilangan Total Pelangi, Shackle, Graf Antiprisma.

1. INTRODUCTION

The development of science and technology to date continues to experience a significant increase. Scientists continue to research to always bring up discoveries that can contribute to science as support for the development of other sciences. One of them is mathematics that scientists discovered hundreds of years ago, which until now can make it easier for humans to solve existing problems. Mathematics is a field of science that can solve or solve problems in real life. One of the branches of mathematics that are often used as an auxiliary sense to describe a problem and theory that is known and widely known today is a graph.

Graph theory first appeared when Leonard Euler tried to solve the Konigsberg bridge problem in 1736. The Konigsberg Bridge is a bridge in an old city in East Prussia which is now commonly called Kalinigrad. Seven bridges were built over the Pregel river to allow residents of Konigsberg to walk from one town to another. Leonhard Euler solved this problem by modeling this problem into a graph[12].

In graph theory there is a topic that is labeling. Graph labeling consists of vertex labeling, edge labeling, and total labeling. One case of labeling is graph coloring. Graph coloring can be used for scheduling, cartography, sudoku, and traffic light settings [14]. Graph coloring is a special case of graph labeling. Graph coloring has three types of coloring: vertex coloring, edge coloring, and region coloring. Spot color indicates the color of a single vertex, with a different color for each of the two adjacent vertex. The color of the side shows the color of the side where the two sides are next to each other and will be a different color. When coloring an area, one area is colored and adjacent areas have a different color [2].

One of the interesting research subjects in coloring on graphs is the rainbow-connection. An edge graph coloured G is rainbow connected if every two vertices are connected by at least one rainbow path in G. For a connected graph G, the rainbow connection number of G, denoted by rc(G), is defined as the smallest number of colours required to make it rainbow connected [4]. Who studies rainbow connected numbers [3] Proper Rainbow Connection Number of Graphs. Next [7] Rainbow Connected Numbers in Corona Operation Result Graphs Antiprism Graphs (AP_m) and Complete Graphs (K4).

Rainbow connected number into total rainbow connected. Total rainbow connected is connected the vertices and edges so that each pair of vertices in the graph has a complete total rainbow path [12] Research on total rainbow connected numbers was first introduced by Uchizawa in 2011, defining k as a natural number. Function If $c : V\,(G) \in E(G) \rightarrow \{1, 2\ldots, k\}$ is said to be a k-coloring -total rainbow on G, each pair of vertex $x, y \in V\,(G)$ has a path called x − y with each edge and each vertex on the path having a different color. Such a path is called the total rainbow path. A graph G is said to be total rainbow connected if for every pair of vertices $x, y \in V(G)$ there is a –total rainbow path $x - y$. The total rainbow connected number is denoted by $trc(G)$. Defined as the minimum number of colors needed to make graph G a total-connected rainbow. Next [1] examines the total rainbow connections by determining the lower and upper bounds for the total rainbow connections number in the amalgamated graph. [13]. Examining total rainbow connected numbers on line graphs and double line graphs of brush graphs.

Total rainbow connected numbers can also be applied to graphs that are the result of operations. The operation used in this research is shackle [5]. Shackle or shackle denoted by shack $(G_1, G_2, \ldots, G_t)$ is a graph resulting from t copies of graph G which is given the symbol (G, t) with $(t \geq 2)$ and t natural numbers every $s, t \in [1, t]$ with $[s - t] \geq 2$ then Gs and Gt do not have the same vertex and for every $i \in [1, n - 1]$, Gt and G(t + 1) have k − 1 such connecting vertex everything is different. If $i \in [1, t]$ Gi is isomorphic to graph H, for shack $(G_1, G_2, \ldots, G_t)$ the shackle of graph H is denoted shack (H, t) [11]. Therefore In this research, we will make rainbow connection number (rc) and total rainbow connection numbers for antiprism graphs with reference to relevant research, using the shackle operation of the AP_3 graph, where $t \geq 2$.

2. MAIN RESULTS

Definition 2.1. *The graph AP_m is an antiprism graph with m vertex size, where $m = 3$. The notated shackle graph of the antiprism $(AP_m, 1, AP_m, 2, \ldots AP_m, t)$graph is the graph resulting from t copies of the graph given the symbol $Shack(AP_3, t)$ for $t \geq 2$ where and t natural numbers. Example (AP_3, t) is a graph G, then the graph is G formed by a set of vertices and a set of edges which are defined as follows:*

$$V(G) = \{v_i, v_{i,1}, v_{i,2}, v_{i,3} \mid i \in [1, t]\} \cup \{u_i \mid i \in [1, t+1]\}$$
$$E(G) = \{u_i u_{i+1} \mid i \in [1, t]\} \cup \{v_i, u_i, v_i u_{i+1} \mid i \in [1, t]\} \cup \{v_{i,j} v_i \mid i \in [1, t], j = \{1,2\} \cup \{v_{i,j} u_i \mid i \in [1, t], j = \{1,3\}\} \cup \{v_{i,j} u_{i+1} \mid i \in [1, t], j = \{2,3\}\} \cup \{v_i = \{1,2\}, v_{i,1} v_{i,2}, v_{i,2} v_{i,3}, v_{i,1} v_{i,3} \mid i = \{1,2\}\}$$

Theorem 2.1. *Let be a positive integer and $t \geq 2$ is a graph G . if $G \cong Shack(AP_3, t)$ then $rc(G) = t + 2$*

Proof. Based on the theorem of Chartrand et al. (2008) it is known that $rc(G) \geq diam(G)$. if it $rc(G) = diam(G)$is enough to show that there is a rainbow path with coloring. $c: E(G) \to \{1, 2, \ldots, rc(G)\}$. Meanwhile, if $rc(G) \neq diam(G)$or $rc(G) > diam(G)$needs to be proven by contradiction. Therefore, to prove Theorem 2.1 it is necessary to show that there is a rainbow path because $rc(G) = diam(G)$.

Given $rc(G) = t + 2$. Next it will be shown that there is a rainbow path that $rc(G) \geq t + 2$. Suppose $rc(G) \leq t + 1$ then there is $c: E(G) \to \{1, 2, \ldots, t + 1\}$. Without compromising generality. For example in the Figure 2.1 the coloring is defined as follows:

$$c(u_i v_i) = c(u_i v_{i+1}) = c(u_i u_{i+1}) = 1 \; for \; i = 1$$

$$c(u_iv_i) = c(u_iv_{i+1}) = c(u_iu_{i+1}) = t + 2 \, for \, i = t$$
$$c(u_iv_i) = c(u_iv_{i+1}) = c(u_iu_{i+1}) = i + 1 \, for \, i = [2, t - 1]$$
$$c(v_{i,1}v_{i,2}) = c(v_{i,1}v_{i,3}) = c\left(v_iv_{i,3}\right) = 1 \, for \, i = 1$$
$$c(v_{i,1}v_{i,2}) = c(v_{i,1}v_{i,3}) = c\left(v_iv_{i,3}\right) = t + 2 \, for \, i = t$$
$$c(v_{i,1}v_{i,2}) = c(v_{i,1}v_{i,3}) = c\left(v_iv_{i,3}\right) = i + 1 \, for \, i = [2, t - 1]$$
$$c(u_iv_{i,1}) = c(u_iv_{i,2}) = c\left(v_iv_{i,2}\right) = t + 1 \, for \, i = [1, t]$$
$$c\left(u_{i+1}v_{i+1,2}\right) = c\left(u_{i+1}v_{i+1,3}\right) = c\left(v_iv_{i,1}\right) = 2 \, for \, i = [1, t]$$

The coloring of the shackle operation results of the antiprism (AP_3) graph is shown in the Figure 2.1

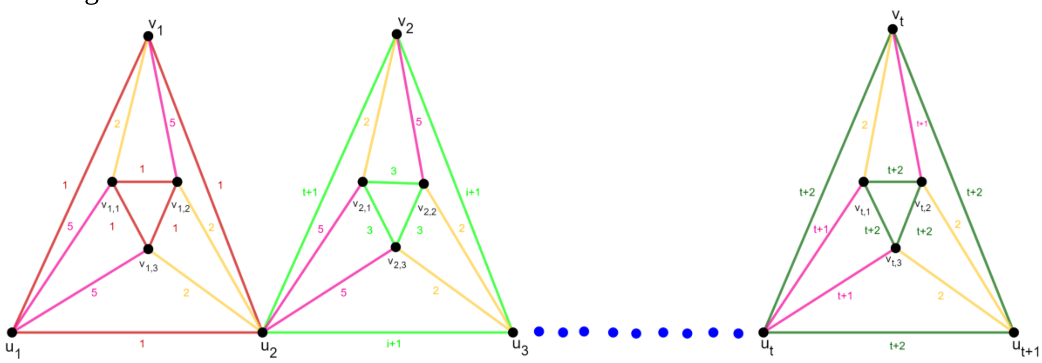

Figure 2.1. Rainbow Coloring (Shack AP_3,t)

So that it is obtained for a rainbow path from the vertex $\in G$ on each side and the vertex of graph G have a different color, so it is called a rainbow path. The rainbow path formed from Figure 2.1 will be shown in the following table in more detail

Table 2.1. Rainbow path

Case	X	Y	Condition	Total Rainbow Path
1	$v_{i,k}$	$v_{t,l}$	$i = [1,2,..t], k = 1, t = t, l$	$v_{i,k}v_{i,k+1}u_{i+1}u_i \dots u_{t,l-1}v_{t,l}$
			$i = [1,2,..t], k = 1, t = t, l$	$v_{i,k}v_{i,k+2}u_{i+2} \dots v_{t,l}$
			$i = [1,2,..t], k = 3, t = t, l$	$v_{i,k}u_i \dots u_tv_{t,l}$
			$i, k = 1, l = 2, t = t - 1$	$v_{i,k}v_{i,k+1}u_{i+1} \dots v_{t,l+1}v_{t,l}$
			$i, k, l = 1, t = t - 1$	$v_{i,k}v_{i,k+2}u_{i+1} \dots v_{t,l}$
			$i, k, l = 1, t = t - 2$	$v_{i,k}u_{i+1}v_{t,l}$

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2	$v_{i,k}$	v_t	$i,k = 1, t = t$	$v_{i,k}v_{i,k+1}u_{i+1}u_{i+2} \dots u_{t-1}v_t$
			$i = 1, k = 2, t = t-1$	$v_{i,k}u_{i+1}v_{i,k+2}v_{i,k}u_{i+2} \dots v_t$
			$i,k = 1, t = t$	$v_{i,k}v_{i,k+1}u_{i+1}u_{i+2} \dots v_t$
			$i = 2, k = 1, t = t-1$	$v_{i,k}v_{i,k+1}u_{i+1} \dots v_t$
			$i = 1, k = 2, t = t-2$	$v_{i,k}u_{i+1}v_t$
3	$v_{i,k}$	u_t	$i,k = 1, t = t+1$	$v_{i,k}v_{i,k+1}u_{i+1}u_{i+2} \dots u_{t-1}u_t$
			$i,k = 1, t = t$	$v_{i,k}u_{i+1}u_{i+2} \dots u_t$
			$i,k = 1, t = t-1$	$v_{i,k}v_{i,k+1}u_{i+1}u_t$
			$i,k = 1, t = t-2$	$v_{i,k}v_{i,k+2}u_t$
4	v_i	u_t	$i = 1, t = t+1$	$v_iu_{i+1}u_{i+2} \dots u_{t-1}u_t$
			$i = 1, t = t$	$v_iu_{i+1}u_{i+2} \dots u_t$
			$i = 1, t = t-2$	$v_iu_{i+1}u_t$
5	v_i	v_t	$i = 1, t = t+1$	$v_iu_{i+1}v_{i+1} \dots u_{t-1}v_t$
			$i = 1, t = t$	$v_iu_{i+1}u_{i+2} \dots u_tv_t$
			$i = 1, t = t-2$	$v_iu_{i+1}v_t$
6	u_i	u_t	$i = 1, t = t+1$	$u_{i,}u_{i+1,}u_{i+2} \dots u_t$
			$i = 1, t = t$	$u_iu_{i+1}u_{i+2} \dots u_t$
			$i = 1, t = t-2$	$u_iu_{i+1}u_t$

Theorem 2.2. *Let t be a positive integer and $t \geq 2$ is a graph G. if $G \cong Shack(AP_3, t)$ then $trc(G) = 2t + 3$.*

Proof. Based on the theorem of Liu et al.(2014) it is known that $trc(G) \geq 2\big(diam(G)\big) - 1$. if it $trc(G) = 2(diam(G)) - 1$ is enough to show that there is a rainbow path with coloring. $c: V(G) \in E(G) \to \{1, 2, \dots, trc(G)\}$. Meanwhile, if $trc(G) > 2(diam(G)) - 1$ needs to be proven by contradiction. Therefore, to prove

Theorem 2.2 it is necessary to show that there is a total rainbow path because $trc(G) \geq 2(diam(G)) - 1$.

Given $trc(G) = 2t + 3$. *Next it will be shown that there is a rainbow path that* $trc(G) \geq 2t + 3$. *Suppose* $trc(G) \leq 2t + 2$ *then there is* $c{:}\,V(G) \in E(G) \rightarrow \{1, 2, \dots, 2t + 2\}$. *Without compromising generality. For example in the figure 2.2 the coloring is defined as follows:*

$$c(u_i v_i) = c(u_i v_{i+1}) = c(u_i u_{i+1}) = t + 2 \; for \; i = 1$$
$$c(u_i v_i) = c(u_i v_{i+1}) = c(u_i u_{i+1}) = 2t + 3 \; for \; i = t$$
$$c(u_i v_i) = c(u_i v_{i+1}) = c(u_i u_{i+1}) = i + t + 2 \; for \; i = [2, t - 1]$$
$$c(v_{i,1} v_{i,2}) = c(v_{i,1} v_{i,3}) = c\left(v_i v_{i,3}\right) = t + 2 \; for \; i = 1$$
$$c(v_{i,1} v_{i,2}) = c(v_{i,1} v_{i,3}) = c\left(v_i v_{i,3}\right) = 2t + 3 \; for \; i = t$$
$$c(v_{i,1} v_{i,2}) = c(v_{i,1} v_{i,3}) = c\left(v_i v_{i,3}\right) = i + t + 2 \; for \; i = [2, t - 1]$$
$$c(u_i v_{i,1}) = c(u_i v_{i,2}) = c\left(v_i v_{i,2}\right) = 2t + 2 \; for \; i = [1, t]$$
$$c\left(u_{i+1} v_{i+1,2}\right) = c\left(u_{i+1} v_{i+1,3}\right) = c\left(v_i v_{i,1}\right) = t + 3 for \; i = [1, t]$$
$$c(v_i) = c(v_{i,1}) = c(v_{i,3}) = 1 \; for \; i = t$$
$$c(v_{i,1}) = 3 \; for \; i = [2, t - 1]$$
$$c(v_{i,2}) = 2 \; for \; i = [2, t - 1]$$
$$c(u_i) = c(u_{i+1}) = (t - 1) + 3 \; for \; i = t$$
$$c(u_i) = c(u_i) = 2 \; for \; i = t$$

The coloring of the shackle operation results of the antiprism (AP_m) graph is shown in the Figure 2.2

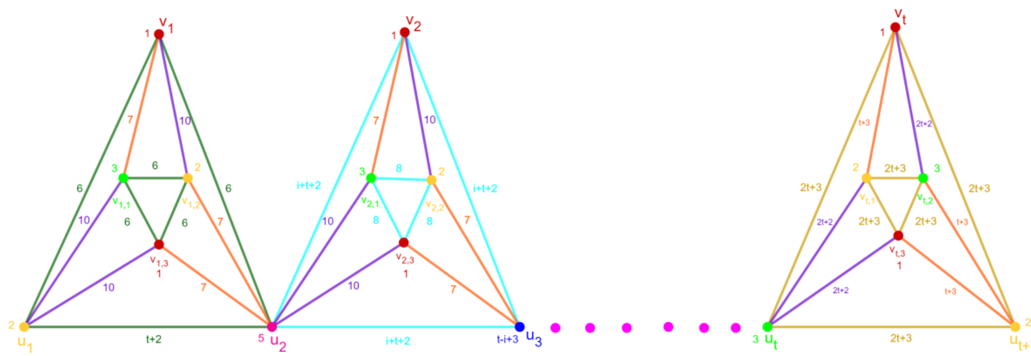

Figure 2.2. Total Rainbow path (shack AP_3, t)

Furthermore, it will be shown that for each x and y at vertex u and there is a v rainbow path $x, y \in V(G)$with $2t + 3$ coloring in the following Table 2.2

Table 2.2. Total Rainbow path

Case	X	Y	Condition	Total Rainbow Path
1	$v_{i,k}$	$v_{t,l}$	$i = [1,2, .. t], k = 1, t = t, l$	$v_{i,k} v_{i,k+1} u_{i+1} u_i \dots u_{t,l-1} v_{t,l}$
			$i = [1,2, .. t], k = 1, t = t, l$	$v_{i,k} v_{i,k+2} u_{i+2} \dots v_{t,l}$

			$i = [1,2,..t], k = 3, t = t, l$	$v_{i,k}u_i \dots u_t v_{t,l}$
			$i, k = 1, l = 2, t = t - 1$	$v_{i,k}v_{i,k+1}u_{i+1} \dots v_{t,l+1}v_{t,l}$
			$i, k, l = 1, t = t - 1$	$v_{i,k}v_{i,k+2}u_{i+1} \dots v_{t,l}$
			$i, k, l = 1, t = t - 2$	$v_{i,k}u_{i+1}v_{t,l}$
2	$v_{i,k}$	v_t	$i, k = 1, t = t$	$v_{i,k}v_{i,k+1}u_{i+1}u_{i+2} \dots u_{t-1}v_t$
			$i = 1, k = 2, t = t - 1$	$v_{i,k}u_{i+1}v_{i,k+2}v_{i,k}u_{i+2} \dots v_t$
			$i, k = 1, t = t$	$v_{i,k}v_{i,k+1}u_{i+1}u_{i+2} \dots v_t$
			$i = 2, k = 1, t = t - 1$	$v_{i,k}v_{i,k+1}u_{i+1} \dots v_t$
			$i = 1, k = 2, t = t - 2$	$v_{i,k}u_{i+1}v_t$
3	$v_{i,k}$	u_t	$i, k = 1, t = t+1$	$v_{i,k}v_{i,k+1}u_{i+1}u_{i+2} \dots u_{t-1}u_t$
			$i, k = 1, t = t$	$v_{i,k}u_{i+1}u_{i+2} \dots u_t$
			$i, k = 1, t = t - 1$	$v_{i,k}v_{i,k+1}u_{i+1}u_t$
			$i, k = 1, t = t - 2$	$v_{i,k}v_{i,k+2}u_t$
4	v_i	u_t	$i = 1, t = t + 1$	$v_i u_{i+1}u_{i+2} \dots u_{t-1}u_t$
			$i = 1, t = t$	$v_i u_{i+1}u_{i+2} \dots u_t$
			$i = 1, t = t - 2$	$v_i u_{i+1}u_t$
5	v_i	v_t	$i = 1, t = t + 1$	$v_i u_{i+1}v_{i+1} \dots u_{t-1}v_t$
			$i = 1, t = t$	$v_i u_{i+1}u_{i+2} \dots u_t v_t$
			$i = 1, t = t - 2$	$v_i u_{i+1}v_t$
6	u_i	u_t	$i = 1, t = t + 1$	$u_{i,}u_{i+1,}u_{i+2} \dots u_t$
			$i = 1, t = t$	$u_i u_{i+1}u_{i+2} \dots u_t$

	$i = 1, t$ $= t - 2$	$u_i u_{i+1} u_t$

Furthermore, for $t \geq 2$, namely by using a coloring proof, it can be shown that each graph shackle has trc$(G) = 2t + 3$ total rainbow coloring.

3. CONCLUSION

The results of this research show that the Shackle operation result of the antiprism graph (AP_3) with $t \geq 2$ $and\ diam(G) = t + 2$. Rainbow connection numbers of shackle product of antiprism graph are obtained $rc(G) = t + 2$. And the total rainbow connection number of shackle product of antiprism graph are obtained $trc(G) = 2t + 3$.

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