JURNAL MATEMATIKA, STATISTIKA DAN KOMPUTASI e-ISSN: 2614-8811
Published by Departement of Mathematics, Hasanuddin University, Indonesia
https://journal.unhas.ac.id/index.php/jmsk/index

Vol. 20, No. 2, January 2024, pp. 360-370
DOI: 10.20956/j.v20i2.32143

r-Chromatic Number On r-Dynamic Vertex Coloring of Comb Graph

Heryati Nur Fatimah Sari[1]*, Budi Nurwahyu[2]*, Jusmawati Massalesse[3]*

**Department of Mathematic, Hasanuddin University, Makassar, Indonesia*
Email: *heryatinurfatimahsari971@gmail.com[1], budinurwahyu@unhas.ac.id[2], jusmawati@gmail.com[3].*

Abstract

Let G be a graph with vertex set $V(G)$ and edge set $E(G)$. An r-dynamic vertex coloring of a graph G is a assigning colors to the vertices of G such that for every vertex v receives at least $min\{r, d(v)\}$ colors in its neighbors. The minimum color used in r-dynamic vertex coloring of graph G is called the r-dynamic chromatic number denoted as $\chi_r(G)$. In this research we well determine the coloring pattern and the r-dynamic chromatic number of the comb graph $P_n \odot K_1$, central graph of comb graph $C(P_n \odot K_1)$, middle graph of comb graph $M(P_n \odot K_1)$, line graph of comb graph $L(P_n \odot K_1)$, sub-division graf of comb graph $S(P_n \odot K_1)$, and para-line graph of comb graph $P(P_n \odot K_1)$.

Keywords: r-Dynamic coloring, chromatic number, comb graph, central graph, middle graph, line graph, sub-division graph, para-line graph.

1. INTRODUCTION AND PRELIMINARIES

Graph theory is a part of discrete mathematics that is used to express model pairwise relations between objects. Graph theory was first introduced in 1736 by a Swiss mathematician named Leonhard Euler. In his writing, Euler provided an illustration of the Königsberg bridge with its four landmasses as points and its seven bridges as sides with research results that it was impossible to cross each of the seven bridges once and return to their original place.

Nowadays, graph theory is starting to experience a lot of development. One of the popular topics in graph theory to study is graph coloring. Graph coloring is divided into three types, including vertex coloring, edge coloring, and region coloring. Furthermore, in graph coloring there is also a concept of r-dynamic coloring. Fierera and Sugeng [3] define r-dynamic coloring, for example, if r is a positive integer, r-dynamic coloring with k-colors on a graph G is the exact coloring of points with k-colors on the graph G such that for every vertex v receives at least $\min\{r, d(v)\}$ colors in its neighbors. The minimum k for graph G in r-dynamic coloring with k-color is called the r-dynamic chromatic number of graph G, denoted as $\chi_r(G)$.

Let G be a simple and finite graph with vertex $V(G)$ and edge set $E(G)$. Comb graph is a graph obtained from corona operations from the path graph P_n with the complete graph K_1. The comb graph is denoted by $P_n \odot K_1$. The central graph [16] $C(G)$ of a graph G is obtained from G by adding an extra vertex on each edge of G, and then joining each pair of vertices of the original graph which were previously non-adjacent. The middle graph [11] of G denoted by $M(G)$, is defined as follows, the vertex set of $M(G)$ is $V(G) \cup E(G)$. Two vertices x, y of $M(G)$ are adjacent in $M(G)$ in case one of the following holds: (i) u, v are in $E(G)$ and u, v are adjacent in G. (ii) $v \in V(G)$, $u \in E(G)$and u, v are incident in G. The line graph [6] of G, denoted by $L(G)$, is the graph whose vertex set is the edge set of G. Two vertices of $L(G)$ are adjacent whenever the corresponding edges of G are adjacent. The sub-division graph $S(G)$ [8] is obtained simply by inserting a new vertex for each edge of G. Para-line graph P(G) [8] is the line graph of a sub-division graph.

Definition 1.1 [8]. An r-dynamic coloring of a graph is a map c from $V(G)$ to the set of colors such that:

(i) if $uv \in (E(G))$, then $c(u) \neq c(v)$, and

(ii) $\forall v \in V(G), |c(N(v))| \geq min\{r, d(v)\}$.

Where r is a positive integer, $N(v)$ denotes the set of all vertices adjacent to v and $d(v)$ its degree.

2. MAIN RESULTS

Proposition and theorems used to find out the r-chromatic number from the r-dynamic coloring of a comb graph and some operations such as central graph, middle graph, line graph, sub-division graph, and para-line graph of the comb graph $P_n \odot K_1$. The proposition and each theorems contain coloring labels which aim to find the minimum color used in r-dynamic vertex coloring of graph.

Proposition 2.1 Let $n \geq 4$, then the r-dynamic chromatic number of the comb graph $P_n \odot K_1$ is:

$$\chi_r(P_n \odot K_1) = \begin{cases} 2, & for\, r = 1 \\ 3, & for\, r = 2 \\ 4, & for\, r = 3 \end{cases}$$

Proof

Let the vertex set and edge set of the comb graph $P_n \odot K_1$ are as follows:

$V(P_n \odot K_1) = \{v_1, v_2, \dots, v_n, u_1, u_2, \dots, u_n\}$ and

$E(P_n \odot K_1) = E(P_n) \cup \{v_i u_i : i = 1,2,3 \dots, n\}$.

Where $E(P_n)$ is the edge set of a path graph.

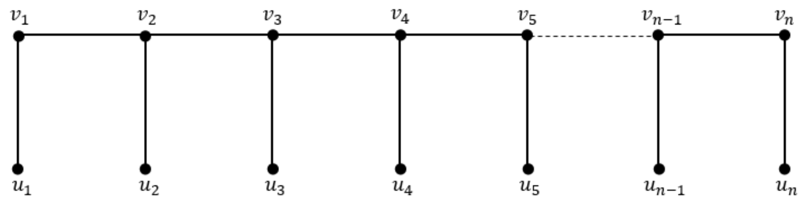

Figure 2.1 Comb Graph $P_n \odot K_1$

The maximum degree of $P_n \odot K_1$ are $\Delta(P_n \odot K_1) = 3$ shown by $v_2, v_3, \dots, v_{n-1}$ and the minimum degree of $P_n \odot K_1$ are $\delta(P_n \odot K_1) = 1$ shown by $u_1, u_2, \dots, u_n$.

Define the mapping $c : V \to Z^+$. Proof is carried out through three cases.

Case 1: For $r = 1$, we use the following colorings as is:

$$c(v_i) = \begin{cases} 1, for\ i\ odd \\ 2, for\ i\ even \end{cases} \text{ and } c(u_i) = \begin{cases} 2, for\ i\ odd \\ 1, for\ i\ even \end{cases}$$

Thus we require 2 colors, that is $\chi_r(P_n \odot K_1) = 2$ for $r = 1$.

Case 2: For $r = 2$, we use the following colorings as is:

$$c(v_i) = \begin{cases} c(v_{3n-2}) = 1 \\ c(v_{3n-1}) = 2 \\ c(v_{3n}) \quad = 3 \end{cases} \text{ and } c(u_i) = \begin{cases} c(u_{3n-2}) = 3 \\ c(u_{3n-1}) = 1 \\ c(u_{3n}) \quad = 1 \end{cases}$$

Thus we require 3 colors, that is $\chi_r(P_n \odot K_1) = 3$ for $r = 2$.

Case 3: For $r = 3$, we use the following colorings as is:

$$c(v_i) = \begin{cases} c(v_{3n-2}) = 2 \\ c(v_{3n-1}) = 3 \\ c(v_{3n}) \quad = 4 \end{cases} \text{ and } c(u_i) = \{1\}$$

Thus we require 4 colors, that is $\chi_r(P_n \odot K_1) = 4$ for $r = 3$.

Theorem 2.1. Let $n \geq 3$, then the r-dynamic chromatic number of the central graph of comb graph $C(P_n \odot K_1)$ is:

$$\chi_r(C(P_n \odot K_1)) = \begin{cases} n, & for\ r = 2 \\ 2n, & for\ 2 \leq r \leq \Delta - 1 \\ 2n+3, & for\ r = \Delta \end{cases}$$

Proof

Let the vertex set and edge set of the central graph of the comb graph $C(P_n \odot K_1)$ are as follows:

$V\big(C(P_n \odot K_1)\big) = \{v_i, u_i, u'_i : 1 \leq i \leq n\} \cup \{v'_i : 1 < i < n-1\}$ and

$E\big(C(P_n \odot K_1)\big) = \{v_i v'_i, v'_i v_{i+1} \mid i = 1,2, n-1\} \cup$
$\{v_i u'_i, u'_i u_i \mid i = 1,2, \dots, n\} \cup \{v_i u_j \mid i \neq j\} \cup$
$\{u_i u_j \mid i = 1,2, \dots, n-1\} \cup \{v_i v_j \mid v_i v_j \notin E(P_n \odot K_1)\}$.

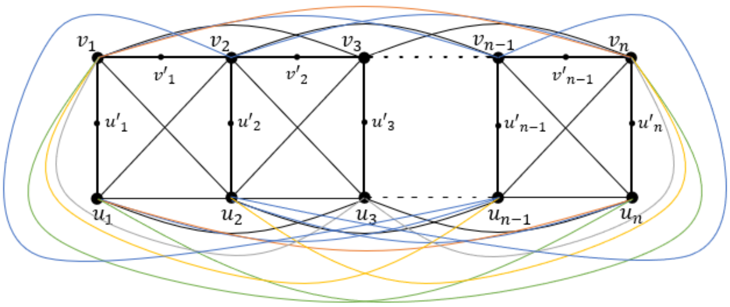

Figure 2.2 Central Graph of Comb Graph $C(P_n \odot K_1)$

The maximum degree of $C(P_n \odot K_1)$ are $\Delta(C(P_n \odot K_1)) = 3$ shown by $v_1, v_2, \dots, v_n, u_1, u_2, \dots, u_n$ and the minimum degree of $C(P_n \odot K_1)$ are $\delta(C(P_n \odot K_1)) = 1$ shown by $v'_1, v'_2, \dots, v'_{n-1}, u'_1, u'_2, \dots, u'_n$.

Define the mapping $c : V \to Z^+$. Proof is carried out through three cases.

Case 1: For $r = 1$, we use the following colorings as is:

$c(v_i) = c(u_i) = i, 1 \leq i \leq n,$

$c(v'_i) = \begin{cases} 4, & for\ i \neq 3,4,\ 1 \leq i \leq n-1 \\ 2, & for\ i = 3,4,\ 1 \leq i \leq n-1 \end{cases}$, and

$c(u'_i) = \begin{cases} 3, & for\ i \neq 3,\ 1 \leq i \leq n \\ 4, & for\ i = 3,\ 1 \leq i \leq n \end{cases}$.

Case 2: For $2 \leq r \leq \Delta - 1$, we use the following colorings as is:

$c(v_i) = i, 1 \leq i \leq n,$

$c(v'_i) = c(u_i) = i + n, 1 \leq i \leq n$, and

$$c(u'_i) = \begin{cases} n, & for\ i = 1, 1 \leq i \leq n-1 \\ i-1, & for\ 2 \leq i \leq n-1 \end{cases}.$$

Case 3: For $r = \Delta$, we use the following colorings as is:

$c(v_i) = i, 1 \leq i \leq n,$

$$c(v'_i) = \begin{cases} 2n+2, & for\ i\ odd \\ 2n+3, & for\ i\ even \end{cases},$$

$c(u_i) = i + n, 1 \leq i \leq n$, and

$c(u'_i) = 2n + 1, 1 \leq i \leq n.$

Theorem 2.2 Let $n \geq 4$, then the *r*-dynamic chromatic number of the middle graph of comb graph $M(P_n \odot K_1)$ is:

$$\chi_r\big(M(P_n \odot K_1)\big) \geq \begin{cases} 4, & for\ 1 \leq r \leq 3 \\ r+1, & for\ 4 \leq r \leq \Delta \end{cases}$$

Proof

Let the vertex set and edge set of the middle graph of the comb graph $M(P_n \odot K_1)$ are as follows:

$V\big(M(P_n \odot K_1)\big) = \{v_i, u_i, u'_i : 1 \leq i \leq n\} \cup \{v'_i : 1 \leq i \leq n-1\}$

$E\big(M(P_n \odot K_1)\big) = \{v_i v'_i \mid i = 1,2, \dots, n-1\} \cup$
$\{v'_i v_{i+1} \mid i = 1,2, \dots, n-1\} \cup \{v_i u'_i \mid i = 1,2, \dots, n\}$
$\{u'_i u_i \mid i = 1,2, \dots, n\} \cup \{v'_i v'_{i+1} \mid i = 1,2, \dots, n-1\}$
$\{v'_i u'_i \mid i = 1,2, \dots, n-1\} \cup \{v'_i u'_{i+1} \mid i = 1,2, \dots, n-1\}.$

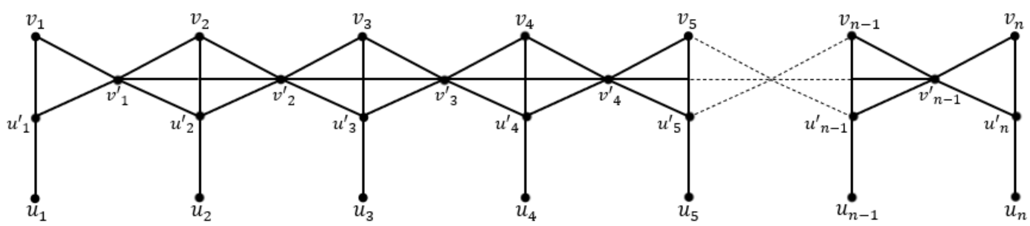

Figure 2.3 Midlle Graph of Comb Graph $M(P_n \odot K_1)$

The maximum degree of $M(P_n \odot K_1)$ are $\Delta(M(P_n \odot K_1)) = 6$ shown by $v'_2, v'_3, \dots, v'_{n-2}$ and the minimum degree of $M(P_n \odot K_1)$ are $\delta(M(P_n \odot K_1)) = 1$ shown by $u_1, u_2, \dots, u_n$.

Define the mapping $c : V \to Z^+$. Proof is carried out through two cases.

Case 1: For $1 \le r \le 3$, we use the following colorings as is:

$$c(v_i) = \begin{cases} 3, for\ i\ odd \\ 4, for\ i\ even \end{cases}, \qquad c(u_i) = 1,$$

$$c(v'_i) = \begin{cases} 2, for\ i\ odd \\ 1, for\ i\ even \end{cases}, \qquad c(u'_i) = \begin{cases} 4, for\ i\ odd \\ 3, for\ i\ even \end{cases}.$$

Case 2: For $4 \le r \le \Delta$;

Subcase (1): for $r = 4$ we use the following colorings as is:

$$c(v_i) = \begin{cases} c(v_{3n-2}) = 1 \\ c(v_{3n-1}) = 2, \\ c(v_{3n}) \quad = 3 \end{cases} \qquad c(u_i) = \begin{cases} r+1, for\ i\ odd \\ r, \quad for\ i\ even \end{cases},$$

$$c(v'_i) = \begin{cases} c(v'_{3n-2}) = 3 \\ c(v'_{3n-1}) = 1, \\ c(v'_{3n}) \quad = 2 \end{cases} \qquad c(u'_i) = \begin{cases} r, \quad for\ i\ odd \\ r+1, for\ i\ even \end{cases}.$$

Subcase (2): for $r = 5$ we use the following colorings as is:

$$c(v_i) = \begin{cases} 2, for\ i\ odd \\ 3, for\ i\ even \end{cases}, \qquad c(u_i) = \begin{cases} r+1,\ for\ i\ odd \\ r, \quad for\ i\ even \end{cases},$$

$$c(v'_i) = \begin{cases} 1, for\ i\ odd \\ 4, for\ i\ even \end{cases}, \qquad c(u'_i) = \begin{cases} r, \quad for\ i\ odd \\ r+1,\ for\ i\ even \end{cases}.$$

Subcase (3): for $r = \Delta = 6$ we use the following colorings as is:

$$c(v_i) = \begin{cases} c(v_{5n-4}) = 4 \\ c(v_{5n-3}) = 1 \\ c(v_{5n-2}) = 5, \\ c(v_{5n-1}) = 2 \\ c(v_{5n}) \quad = 3 \end{cases} \qquad c(v'_i) = \begin{cases} c(v'_{5n-4}) = 2 \\ c(v'_{5n-3}) = 3 \\ c(v'_{5n-2}) = 4, \\ c(v'_{5n-1}) = 1 \\ c(v'_{5n}) \quad = 5 \end{cases}$$

$$c(u_i) = \begin{cases} r+1, for\ i\ odd \\ r, \quad for\ i\ even \end{cases}, \qquad c(u'_i) = \begin{cases} r, \quad for\ i\ odd \\ r+1,\ for\ i\ even \end{cases}.$$

Theorem 2.3 Let $n \ge 4$, then the r-dynamic chromatic number of the line graph of comb graph $L(P_n \odot K_1)$ is:

$$\chi_r(L(P_n \odot K_1)) = \begin{cases} 3, & for\ 1 \le r \le 2 \\ 4, & for\ r = 2 \\ 5, & for\ r = 4 \end{cases}$$

Proof

Let the vertex set and edge set of the line graph of the comb graph $L(P_n \odot K_1)$ are as follows:

$V\big(L(P_n \odot K_1)\big) = \{w_1, w_2, w_3, \dots, w_n, w'_1, w'_2, w'_3, \dots, w'_{n-1}\}$ dan

$E\big(L(P_n \odot K_1)\big) = \{w'_i w'_{i+1} \mid i = 1,2,\dots, n-2\} \cup \{w'_i w_i \mid i = 1,2,\dots, n-1\} \cup$
$\{w'_i w_{i+1} \mid i = 1,2,\dots, n-1\}$.

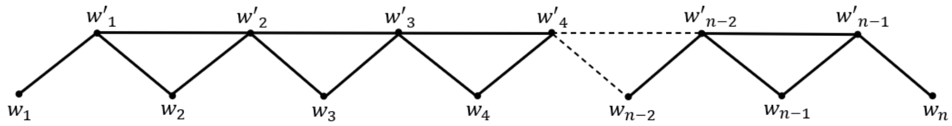

Figure 2.4 Line Graph of Comb Graph $L(P_n \odot K_1)$

The maximum degree of $L(P_n \odot K_1)$ are $\Delta(L(P_n \odot K_1)) = 4$ shown by $w_2, w_3, \dots, w_{n-2}$. and the minimum degree of $L(P_n \odot K_1)$ are $\delta(L(P_n \odot K_1)) = 1$ shown by w_1, and w_n.

Define the mapping $c : V \to Z^+$. Proof is carried out through three cases.

Case 1: For $1 \le r \le 2$, we use the following colorings as is:

$c(w_i) = 1$, $c(w'_{2i-1}) = 2$, and $c(w'_{2i}) = 3$.

Case 2: For $r = 3$, we use the following colorings as is:

$c(w_i) = \begin{cases} 1, for\ i\ odd \\ 3, for\ i\ even \end{cases}$, and $c(w'_i) = \begin{cases} 2, for\ i\ odd \\ 4, for\ i\ even \end{cases}$.

Case 3: For $r = \Delta$, we use the following colorings as is:

$c(w_i) = \begin{cases} 1, for\ i\ odd \\ 3, for\ i\ even \end{cases}$, and $c(w'_i) = \begin{cases} c(w_{3n-2}) = 2 \\ c(w_{3n-1}) = 4. \\ c(w_{3n}) \quad = 5 \end{cases}$

Theorem 2.4 Let $n \ge 3$, then the *r*-dynamic chromatic number of the subdivision graph of comb graph $S(P_n \odot K_1)$ is:

$$\chi_r\big(S(P_n \odot K_1)\big) = r + 1, 1 \le r \le 3$$

Proof

Let the vertex set and edge set of the subdivision graph of the comb graph $S(P_n \odot K_1)$ are as follows:

$V\big(S(P_n \odot K_1)\big) = \{v_i, u_i, u'_i : 1 \le i \le n\} \cup \{v'_i : 1 \le i \le n-1\}$ and
$E\big(S(P_n \odot K_1)\big) = \{v_i v'_i, v'_i v_{i+1} \; ; \; 1 \le i \le n-1\} \cup \{v_i u'_i, u'_i u_i \; ; \; 1 \le i \le n\}$.

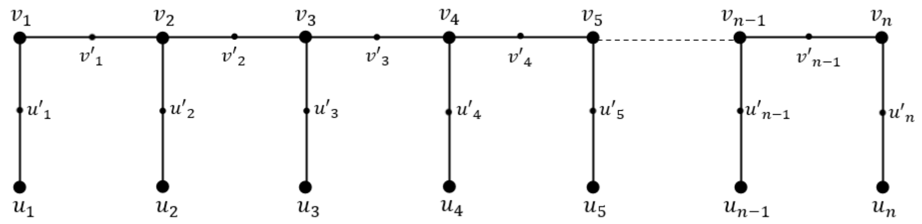

Figure 2.5 Subdivision Graph of Comb Graph $S(P_n \odot K_1)$

The maximum degree of $S(P_n \odot K_1)$ are $\Delta(S(P_n \odot K_1)) = 3$ shown by $v_2, v_3, \dots, v_{n-1}$. and the minimum degree of $S(P_n \odot K_1)$ are $\delta(S(P_n \odot K_1)) = 1$ shown by $u_1, u_2, \dots, u_n$.

Define the mapping $c : V \to Z^+$. Proof is carried out through three cases.

Case 1: For $r = 1$, we use the following colorings as is:

$c(v_i) = 1, c(v'_i) = 2, c(u_i) = 1$, and $c(u'_i) = 2$.

Case 2: For $r = 2$, we use the following colorings as is:

$c(v_i) = \begin{cases} 1, for\ i\ odd \\ 2, for\ i\ even \end{cases}, c(v'_i) = 3, c(u_i) = 3$, and $c(u'_i) = \begin{cases} 2, for\ i\ odd \\ 1, for\ i\ even \end{cases}$

Case 3: For $r = 3$, we use the following colorings as is:

$c(v_i) = \begin{cases} 1, for\ i\ odd \\ 2, for\ i\ even \end{cases}$, $\quad c(v'_i) = \begin{cases} 3, for\ i\ odd \\ 4, for\ i\ even \end{cases}$

$c(u_i) = 4$, and $\quad c(u'_i) = \begin{cases} 2, for\ i\ odd \\ 1, for\ i\ even \end{cases}$.

Theorem 2.5 Let $n \ge 3$, then the *r*-dynamic chromatic number of the para-line graph of comb graph $P(P_n \odot K_1)$ is:

$$\chi_r\big(P(P_n \odot K_1)\big) = \begin{cases} 3, & for\ 1 \le r \le 2 \\ 4, & for\ r = \Delta \end{cases}$$

Proof

Let the vertex set and edge set of the para-line graph of the comb graph $P(P_n \odot K_1)$ are as follows:

$V\big(P(P_n \odot K_1)\big) = \{w_i\,; 1 \le i \le 2n\} \cup \{w'_i\,; 1 \le i \le 2n-2\}$ and

$E\big(P(P_n \odot K_1)\big) = \{w_i w_{i+1}\,; 1 \le i \le 2n\} \cup \{w'_i w_i \,| i = 1,3,5,\dots,2n-3\} \cup$
$\{w'_{i+1} w_{i+2}\,;\ 1 \le i \le 2n-3\} \cup \{w'_i w'_{i+1}\,; 1 \le i \le 2n-2\}$.

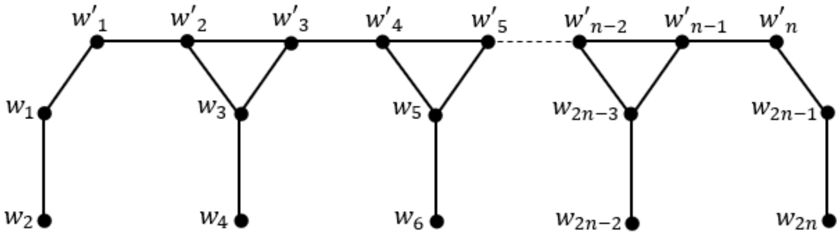

Figure 2.5 Subdivision Graph of Comb Graph $S(P_n \odot K_1)$

The maximum degree of $P(P_n \odot K_1)$ are $\Delta(P(P_n \odot K_1)) = 3$ shown by $w'_2, w'_3, \dots, w'_{n-1}$ and $w'_3, w'_5, \dots, w'_{2n-3}$, and the minimum degree of $P(P_n \odot K_1)$ are $\delta(P(P_n \odot K_1)) = 1$ shown by $w_2, w_4, \dots, w_{2n}$.

Define the mapping $c : V \to Z^+$. Proof is carried out through two cases.

Case 1: For $1 \le r \le 2$, we use the following colorings as is:

$c(w_i) = \begin{cases} 2, for\ i\ odd \\ 1, for\ i\ even \end{cases}$, and $c(w'_i) = \begin{cases} 1, for\ i\ odd \\ 3, for\ i\ even \end{cases}$.

Case 2: For $r = \Delta$, we use the following colorings as is:

$c(w_i) = \begin{cases} c(w_{2i-1}) = 4 \\ c(w_{6i-6}) = 2 \\ c(w_{6i-2}) = 1 \\ c(w_{6i}) \quad = 3 \end{cases}$ and $c(w'_i) = \begin{cases} c(w'_{3i-2}) = 1 \\ c(w'_{3i-1}) = 2. \\ c(w_{3i}) \quad = 3 \end{cases}$

3. CONCLUSION

In this paper we have shown that comb graphs $P_n \odot K_1$ and some of their operations such as central graph, middle graph, line graph, subdivision graph, and para-line of the comb graph $P_n \odot K_1$ are r-dynamic graphs. The *r*-dynamic chromatic numbers of each graph are:

$P_n \odot K_1$:

$$\chi_r(P_n \odot K_1) = \begin{cases} 2, & for\ r = 1 \\ 3, & for\ r = 2 \\ 4, & for\ r = 3 \end{cases}$$

$C(P_n \odot K_1)$:

$$\chi_r(C(P_n \odot K_1)) = \begin{cases} n, & for\ r = 2 \\ 2n, & for\ 2 \le r \le \Delta - 1 \\ 2n+3, & for\ r = \Delta \end{cases}$$

$M(P_n \odot K_1)$:

$$\chi_r(M(P_n \odot K_1)) = \begin{cases} 4, & for\ 1 \le r \le 3 \\ r+1, & for\ 4 \le r \le \Delta \end{cases}$$

$L(P_n \odot K_1)$:

$$\chi_r(L(P_n \odot K_1)) = \begin{cases} 3, & for\ 1 \le r \le 2 \\ 4, & for\ r = 2 \\ 5, & for\ r = 4 \end{cases}$$

$S(P_n \odot K_1)$:

$$\chi_r(S(P_n \odot K_1)) = r + 1, 1 \le r \le 3$$

$P(P_n \odot K_1)$:

$$\chi_r(P(P_n \odot K_1)) = \begin{cases} 3, & for\ 1 \le r \le 2 \\ 4, & for\ r = \Delta \end{cases}$$

REFERENCES

[1] Alishahi, M., 2012. *Dynamic Chromatic Number of Regular Graphs. Discrete Applied Mathematics*, 160, 2098–2103.

[2] Alfian, Y. H., Ika. H. A., and Dafik, 2015. Pewarnaan Titik Pada Operasi Graf Sikel dengan Graf Lintasan. CGANT Universitas Jember

[3] Audi Fierera and Kiki A. Sugeng, 2021. Pewarnaan Simpul *r*-Dinamis pada Graf Teratai T_n. Ambon, Universitas Pattimura

[4] Chartrand, G., dan. Zhang.P., 2009. *Chromatic Graph Theory of a Graph*. USA: CRC Press

[5] Frucht, R., dan Harary, F., 1970. *On The Corona of Two Graphs*. Vol. 4, Santa Maria University dan University of Michigan

[6] Harary, F., 1969. *Graph Theory*, Narosa Publishing Home, New Delhi.

[7] Hasmawati. 2020. Pengantar dan Jenis-Jenis Graf. UPT Unhas Press

[8] Kalaiselvi, K., Mohanapriya, and Vernold, V.J., 2021. *On r-Dynamic Coloring of Comb Graph*. Vol. 27, 2021, No. 2, 191–200

[9] Lai, H. J., and Montgomery, B., 2001. *Dynamic Coloring of Graphs*. West Virginia University, Morgantown

[10] Lai, H. J., Montgomery, B., Poon, H., 2003. *Upper Bounds of Dynamic Chromatic Number*. Ars Combinatoria 68

[11] Michalak, D., 1983. *On Middle and Total Graphs with Coarseness Number Equal 1.* Lecture Notes in Mathematics, 1018, Springer Verlag Graph Theory, Lagow proceedings, Berlin Heidelberg, New York, Tokyo, 139–150.

[12] Muhammad,D.T., 2015. Nilai Kromatik dan Pewarnaan Titik *r* Dinamis pada Graf Khusus dan Shakel. Jember: Universitas Jember

[13] Novian N.F., 2015. Pewarnaan Titik dan Sisi *r*-Dinamis Pada Graf Hasil Operasi Comb Sisi. Jombang.

[14] Slamin, 2009. Desain Jaringan Pendekatan Teori Graf. Jember: Universitas Jember.

[15] Taherkhani, A., 2016. *r-Dynamic Chromatic Number of Graphs. Discrete Applied Mathematics*, 201, 222-227

[16] Vernold V. J., 2007. *Harmonious Coloring of Total Graphs, n-leaf, Central Graphs and Circumdetic Graphs*, Ph.D. Thesis, Bharathiar University, Coimbatore, India.