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The Comparison of Inverse Gaussian and Gamma Regression: Application on Stunting Data in Jepara

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Abstract

Many research data have distributions other than the normal distribution, called exponential family distributions. The exponential family of distributions includes the inverse Gaussian and Gamma distributions. There are parallels between these two distributions in terms of the kind of random variable and how well they work. Finding the optimal model using inverse Gaussian and Gamma regression on stunting data in Jepara is the goal of this study. Maximum Likelihood Estimation is used for parameter estimation, Maximum Likelihood Ratio Test is used for simultaneous parameter testing, and Wald testing is used for partial parameter testing. For this case, the best model is inverse Gaussian regression. Exclusive breastfeeding, low birth weight babies, clean drinking water facilities, and the number of Integrated Service Post (Posyandu) influence the percentage of stunting in Jepara..

Keywords: inverse Gaussian regression, Gamma regression, Maximum Likelihood Estimation, Maximum Likelihood Ratio Test

1. INTRODUCTION

Instead of being dispersed normally, the majority of study data in the field follows is exponential family [13]. In statistics, there are quite a lot of distributions that fall into the exponential family. Bernoulli, Binomial, Negative Binomial, Poisson, Normal, Exponential, Gamma, Inverse Gaussian, and Weibull distributions are members of the exponential family.

Inverse Gaussian and Gamma distribution are two members of the exponential family. The two distributions are similar but the Inverse Gaussian plot is more slanted and the plot is sharper [10] so usually data that follows the Gamma distribution can also follow the inverse Gaussian. There are many similarities to the Inverse Gaussian and Gamma distributions, using non-negative continuous random variables at intervals, being an Exponential family, having two parameters (for Gamma two parameters), and a linear model can be formed using the Generalized Linear Model

(GLM) [17]. With GLM, a link function is used to connect the expectations of the response variable with the explanatory variables in the linear model [11]. The choice of link function type depends on the ease of estimating model parameters. When Inverse Gaussian and Gamma are formed in the regression model, the types of link functions in this research are log and canonical functions. Log function for Inverse Gaussian regression and canonical for Gamma regression. The two link functions were chosen because these functions are often used in research [6]. The two distributions are similar but research using and comparing the two is limited. Through this research, inverse Gaussian and Gamma regression will be modeled and compared to the stunting data.

In 2020, Jayalath and Chhikara [7] conducted a survival analysis using an inverse Gaussian distribution, which helped them develop a Gibbs sampling technique. Compared to Inverse Gaussian, there are more techniques used in survival for the Gamma distribution. Kiche et al. [9] renamed the Gamma distribution as the Generalized Gamma (GG) distribution. The application of GG to survival data is correct, except that GG uses three parameters. Likewise Cox et al. [5] also examined GG in survival analysis. We can see that the majority of Inverse Gaussian and Gamma applications are in the field of survival, such as a patient's survival against certain diseases, so this research is relevant to apply to stunting cases.

Stunting is a condition of a toddler experiences a lack of nutritional intake for a long period of time so that the child experiences growth disorders, a height shorter than the age standard [8] The human growth can occur in an existing fetus in the womb and only becomes visible when the child is two years old [14]. Data on the percentage of stunting in Jepara Regency from 2019-2022 there has been an increase. According to the Indonesian Nutrition Status Survey (SSGI), the prevalence of stunting in Jepara 18.2% in 2022, decreased significantly compared to the previous year by 25% and the number of stunted babies in Jepara continues to decline from year to year. Therefore, an analysis is needed to determine the factors causing stunting in Jepara in order to they can be addressed appropriately. One model to find factors causing stunting in Jepara is inverse Gaussian and Gamma regression.

2. LITERATURE REVIEW

2.1 Inverse Gaussian Distribution

The Inverse Gaussian distribution, also known as the Wald distribution, is a family of two-parameter exponentials with non-negative continuous random variables on the interval. The Inverse Gaussian distribution was first studied by Schrowdinger and Smoluchowski in relation to Brownian motion [15]. The name Inverse Gaussian was given by Tweedie in 1956. Suppose Y a continuous random variable is said to have an Inverse Gaussian distribution with location parameters μ and scale parameters λ if it satisfies the probability density function:

$$f(y;\mu,\lambda)=\begin{cases}\left(\dfrac{\lambda}{2\pi y^3}\right)^{1/2}\exp\left(-\dfrac{\lambda(y-\mu)^2}{2\mu^2 y}\right), & y>0,\ \mu>0\\ 0 & ,y \text{ others}\end{cases} \tag{1}$$

2.2 Gamma Distribution

The Gamma distribution is often used in probability models for waiting times. Like the Inverse Gaussian distribution, the random variable in the Gamma distribution is also positively continuous. The continuous random variable Y has a Gamma distribution with two parameters α and β, which have a probability density function

$$f(y)=\begin{cases}\frac{1}{\beta^\alpha\Gamma(\alpha)}y^{\alpha-1}e^{-y/\beta} & ,\ y>0\\ 0 & ,y\ others\end{cases} \tag{2}$$
$$\alpha>0,\beta>0$$

The link function commonly used in Gamma regression is the canonical link [12]. The method for determining parameter estimates and parameter testing is the same as in Inverse Gaussian regression.

2.3 Generalized Linear Model

Not all cases that occur in the field contain normally distributed responses. Some of the responses that occur have more general distributions such as Gamma and Inverse Gaussian. To handle conditions where the responses are not normally distributed but are still independent of each other, statisticians pioneered by Nelder and Wedderburn in 1972 have developed a linear model known as the Generalized Linear Model (GLM) or generalized linear model. This linear model uses the assumption that the response has an exponential family distribution [17].

According to McCullagh and Nelder (1989) [11], we assume that each component of Y has an exponential family distribution, so the form of the probability density function is as follows:

$$f_Y(y;\theta,\phi)=\exp\left\{\frac{y\,\theta-b(\theta)}{a(\phi)}+c(y,\phi)\right\}, \tag{3}$$

where θ is a canonical parameter, ϕ dispersion parameter, and $a(\phi),b(\theta),c(y,\phi)$ are a known function. There are three important components in the generalized linear model. They are

1. The random components of Y are independently distributed in the exponential family
2. The systematic component is a linear predictor η written with an equation

$$\eta = \sum_{j=1}^{k} x_j \beta_j \quad , j = 1,2,...,k \tag{4}$$

3. The link function $g(.)$ that connects the random component in this case is the mean with the systematic component is called the linear predictor.

2.4 Goodness of fit test

Many hypothesis testing for goodness of fit likes Anderson Darling and Variance Ratio. Villasenor and Gonzales-Estrada [18] stated that the variance ratio test is more powerful than the Anderson Darling test. Furthermore, Variance Ratio is used in this study. Hypothesis is given below

H_0 : the data fits the hypothesized distribution

H_1 : the data does not fit the hypothesized distribution

The formula of Variance ratio is given as follows

$$F = \frac{S_1^2}{S_2^2} \tag{5}$$

H_0 is rejected when $F \geq F_{\alpha,n-k}$ with n the number of samples and k the number of parameters.

2.5 Multicollinearity

Multicollinearity is an assumption that requires predictor variables to be independent of each other. It can be indicated by a VIF value of less than 10 [4], [16]. VIF formula is given below

$$VIF_j = \frac{1}{1-R_j^2} \tag{6}$$

with R_j^2 being the coefficient of determination of variable-*j*.

2.6 Maximum Likelihood Estimation

Maximum Likelihood Estimation (MLE) is parameter estimation that yields unbiased parameters [3]. Wang et al. [19] employ MLE with maximum ranked set sampling for inverse Gaussian statistical inference. The uneven samples are used in the sampling operation to reduce errors and increase efficiency. Estimation starts from forming a likelihood function from the probability density function. The likelihood function is defined as follows [3]

$$L(\theta) = f(x_1,\theta)......f(x_n,\theta). \tag{7}$$

hen determining the ln likelihood function. The next stage is determining the ln likelihood function and finding derivative of each parameter. if the result is not closed form then continued with numerical iteration.

2.7 Parameter Testing

Parameter testing was carried out twice, they are simultaneous and partial parameter testing. Simultaneous parameter testing using Maximum Likelihood Ratio Test (MLRT) with the following hypothesis

Hypothesis:

H_0 : all parameters are significant simultaneously

H_1 : at least one parameter is not significant simultaneously

Test statistics of MLRT is given $G^2 = -2\ln\Lambda$ with

$$\Lambda = \left(\frac{L\left(\hat{\omega}\right)}{L\left(\hat{\Omega}\right)}\right) = \left(\frac{\prod_{i=1}^{n}\left(\frac{\hat{\lambda}_\omega}{2\pi y_i^3}\right)^{1/2} \exp\left(\frac{-\hat{\lambda}_\omega\left(y_i - e^{\hat{\beta}_0}\right)^2}{2\left(e^{\hat{\beta}_0}\right)^2 y_i}\right)}{\prod_{i=1}^{n}\left(\frac{\hat{\lambda}}{2\pi y_i^3}\right)^{1/2} \exp\left(\frac{-\hat{\lambda}\left(y_i - e^{\mathbf{x}_i^T\hat{\boldsymbol{\beta}}}\right)^2}{2\left(e^{\left(\mathbf{x}_i^T\hat{\boldsymbol{\beta}}\right)}\right)^2 y_i}\right)}\right) \tag{8}$$

Wilks [20] represent $-2\ln\Lambda$ is a likelihood ratio for *n* the number of parameters under H_1 and H_0 approaches infinity, which asymptotically yields a Chi-square distribution with degrees of freedom. Reject H_0 if $G^2{}_{\text{hitung}} > \chi^2_{k,\alpha}$.

2.8 Model Selection Criteria

Among the factors used to determine which model is best is the application of Akaike's Information Criterion (AIC). The Maximum Likelihood Estimation (MLE) method serves as the foundation for this approach and produces unbiased parameter [2]. One benefit of using AIC is that it already has a large number of predictor variables, which is very helpful for choosing the optimum regression model. The AIC formula, according to [1] looks like this:

$$\text{AIC} = -2\ln L(.) + 2k \tag{9}$$

with $L(.)$: likelihood function

k : number of parameters

The best model is the model with the smallest AIC value.

3. METHODS

This study uses inferential analysis, specially inverse Gaussian and Gamma regression. The two models are applied on stunting data in Jepara as response variable and exclusive breastfeeding, low birth weight babies, clean drinking water facilities, the number of posyandu as predictor variables. The data used is secondary data from the Central Statistics Agency in 2022. The following are the analysis stages in this research

1) Parameter estimation of inverse Gaussian and Gamma regression
2) Inverse Gaussian and gamma regression modeling on stunting data in Jepara
3) Simultaneous and partial parameter testing
4) Interpretation of result
5) Determining the best model based on AIC criteria

4. RESULTS

4.1 Estimation of Inverse Gaussian and Gamma Parameters

In this study, Maximum Likelihood Estimation was used to estimate parameters for Inverse Gaussian and Gamma regression. Firstly, we estimate parameters of Inverse Gaussian regression with determine probability density function. Suppose n sample random $y_1, y_2, ..., y_n$ so that we obtain probability density function of inverse Gaussian

$$f(y_i) = \left(\frac{\lambda}{2\pi y_i^3}\right)^{1/2} \exp\left(-\frac{\lambda(y_i - \mu_i)^2}{2\mu_i^2 y_i}\right) \quad ,\mu_i > 0,\ \lambda > 0,\ y_i > 0. \tag{10}$$

With log link function, $\mu_i = \exp(\beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + ... + \beta_k x_{ik}) = \exp(\mathbf{x}_i^T \boldsymbol{\beta})$ we get following the new formula

$$f(y_i) = \left(\frac{\lambda}{2\pi y_i^3}\right)^{1/2} \exp\left(\frac{-\lambda\left(y_i - \exp\left(\mathbf{x}_i^T\boldsymbol{\beta}\right)\right)^2}{2\left(\exp\left(\mathbf{x}_i^T\boldsymbol{\beta}\right)\right)^2 y_i}\right) \tag{11}$$

Secondly, we determine likelihood function from (2) as the first step of Maximum Likelihood Estimation.

$$L(\boldsymbol{\beta}, \lambda | y_1, y_2, ..., y_n) = \prod_{i=1}^{n} \left\{ \left(\frac{\lambda}{2\pi y_i^3}\right)^{1/2} \exp\left(\frac{-\lambda\left(y_i - \exp\left(\mathbf{x}_i^T\boldsymbol{\beta}\right)\right)^2}{2\left(\exp\left(\mathbf{x}_i^T\boldsymbol{\beta}\right)\right)^2 y_i}\right) \right\} \tag{12}$$

Thirdly, we determine the ln-likelihood function from (12).

$$\ln L(\boldsymbol{\beta}, \lambda | y_1, y_2, ..., y_n) = \ln\left(\prod_{i=1}^{n} \left\{ \left(\frac{\lambda}{2\pi y_i^3}\right)^{1/2} \exp\left(\frac{-\lambda\left(y_i - \exp\left(\mathbf{x}_i^T\boldsymbol{\beta}\right)\right)^2}{2\left(\exp\left(\mathbf{x}_i^T\boldsymbol{\beta}\right)\right)^2 y_i}\right) \right\} \right)$$

$$= \frac{n}{2}\ln\lambda - \frac{n}{2}\ln(2\pi) - \frac{3}{2}\sum_{i=1}^{n}\ln y_i - \frac{\lambda}{2}\sum_{i=1}^{n}\left(\frac{\left(y_i^2 - 2y_i e^{\left(\mathbf{x}_i^T\boldsymbol{\beta}\right)} + \left(e^{\left(\mathbf{x}_i^T\boldsymbol{\beta}\right)}\right)^2\right)}{\left(e^{\left(\mathbf{x}_i^T\boldsymbol{\beta}\right)}\right)^2 y_i} \right) \tag{13}$$

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The estimator can be obtained by determining the first derivative of the ln-likelihood function for each parameter. The first partial derivative of $\boldsymbol{\beta}^T$ as follows:

$$\frac{\partial \ln L\left(\boldsymbol{\beta},\lambda \middle| y_1,y_2,...,y_n\right)}{\partial \boldsymbol{\beta}^T} = \lambda \sum_{i=1}^{n} \frac{\mathbf{x}_i y_i}{\left(e^{\left(\mathbf{x}_i^T \boldsymbol{\beta}\right)}\right)^2} - \lambda \sum_{i=1}^{n} \frac{\mathbf{x}_i}{e^{\left(\mathbf{x}_i^T \boldsymbol{\beta}\right)}} \quad (14)$$

In addition, the first partial derivative of λ is

$$\frac{\partial \ln L\left(\boldsymbol{\beta},\lambda \middle| y_1,y_2,...,y_n\right)}{\partial \lambda} = \frac{n}{2\lambda} - \frac{1}{2} \sum_{i=1}^{n} \frac{y_i}{\left(e^{\left(\mathbf{x}_i^T \boldsymbol{\beta}\right)}\right)^2} + \sum_{i=1}^{n} \frac{1}{e^{\left(\mathbf{x}_i^T \boldsymbol{\beta}\right)}} - \frac{1}{2} \sum_{i=1}^{n} \frac{1}{y_i}. \quad (15)$$

The results of partial derivative $\boldsymbol{\beta}$ dan λ still contain other parameters so it is necessary to continue the numerical calculation with Fisher scoring iteration. Fisher Scoring was chosen because its convergence is guaranteed [13],[15],[21]. The followings are Fisher Scoring logarithm.

1. Determine an initial value $\boldsymbol{\gamma}_0$ with $\boldsymbol{\gamma} = \begin{bmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \\ \vdots \\ \beta_k \\ \lambda \end{bmatrix}$ at iteration $m = 0$

2. Determine the gradient matrix $\mathbf{g}\left(\boldsymbol{\gamma}_m\right) = \begin{bmatrix} \dfrac{\partial \ln L\left(\boldsymbol{\beta},\lambda \middle| y_1,y_2,...,y_n\right)}{\partial \boldsymbol{\beta}^T} \\ \dfrac{\partial \ln L\left(\boldsymbol{\beta},\lambda \middle| y_1,y_2,...,y_n\right)}{\partial \lambda} \end{bmatrix}$

3. Discover the Fisher information matrix $I\left(\mathbf{H}\left(\gamma_m\right)\right) = -E\left(\mathbf{H}\left(\gamma_m\right)\right)$ with

4. $$\mathbf{H}\left(\boldsymbol{\gamma}_m\right) = \begin{bmatrix} \dfrac{\partial^2 \ln L\left(\boldsymbol{\beta},\lambda \middle| y_1,y_2,...,y_n\right)}{\partial \boldsymbol{\beta}^T \partial \boldsymbol{\beta}} & \dfrac{\partial^2 \ln L\left(\boldsymbol{\beta},\lambda \middle| y_1,y_2,...,y_n\right)}{\partial \boldsymbol{\beta}^T \partial \lambda} \\ \dfrac{\partial^2 \ln L\left(\boldsymbol{\beta},\lambda \middle| y_1,y_2,...,y_n\right)}{\partial \lambda \partial \boldsymbol{\beta}^T} & \dfrac{\partial^2 \ln L\left(\boldsymbol{\beta},\lambda \middle| y_1,y_2,...,y_n\right)}{\partial \lambda^2} \end{bmatrix}$$

4. Calculate $\gamma_{m+1} = \gamma_m + \left(I\left(H\left(\gamma_m\right)\right)\right)^{-1} g\left(\gamma_m\right)$

5. If $\left|\gamma_{m+1} - \gamma_m\right| \le \varepsilon$ iteration stops and when $\left|\gamma_{m+1} - \gamma_m\right| > \varepsilon$ so return step 2 with $m = m+1$

Furthermore, the Gamma regression parameter estimation is carried out using MLE as follows

$$L(\alpha,\boldsymbol{\beta}) = \prod_{i=1}^{n} f(y_i)$$
$$= \prod_{i=1}^{n} \left[\frac{1}{\Gamma(\alpha)\left(\frac{\exp(\mathbf{x}_i^T\boldsymbol{\beta})}{\alpha}\right)^{\alpha}} y^{\alpha-1} \exp\left(\frac{-\alpha y_i}{\exp(\mathbf{x}_i^T\boldsymbol{\beta})}\right) \right]$$

From the partial derivative of parameter α is obtained as follows

$$\frac{\partial \ln L(\alpha,\boldsymbol{\beta})}{\partial \alpha} = -n\frac{1}{\Gamma(\alpha)}\Gamma(\alpha)' - \sum_{i=1}^{n}\mathbf{x}_i^T\boldsymbol{\beta} + n\ln(\alpha) + n + \sum_{i=1}^{n}\frac{-y_i}{\exp(\mathbf{x}_i^T\boldsymbol{\beta})} \tag{16}$$

Meanwhile, the partial derivative of parameter $\boldsymbol{\beta}$ is

$$\frac{\partial \ln L(\alpha,\boldsymbol{\beta})}{\partial \boldsymbol{\beta}^T} = -\alpha\sum_{i=1}^{n}\mathbf{x}_i + \alpha\sum_{i=1}^{n}\frac{x_i y_i}{\exp(\mathbf{x}_i^T\boldsymbol{\beta})} \tag{17}$$

As with inverse Gaussian regression, estimating Gamma parameters is also followed by Fisher scoring with the same algorithm. The difference is the Fisher information matrix

$$\mathbf{H}(\boldsymbol{\gamma}_m) = \begin{bmatrix} \frac{\partial^2 \ln L(\boldsymbol{\beta},\alpha|y_1,y_2,...,y_n)}{\partial\boldsymbol{\beta}^T\partial\boldsymbol{\beta}} & \frac{\partial^2 \ln L(\boldsymbol{\beta},\alpha|y_1,y_2,...,y_n)}{\partial\boldsymbol{\beta}^T\partial\alpha} \\ \frac{\partial^2 \ln L(\boldsymbol{\beta},\alpha|y_1,y_2,...,y_n)}{\partial\alpha\partial\boldsymbol{\beta}^T} & \frac{\partial^2 \ln L(\boldsymbol{\beta},\alpha|y_1,y_2,...,y_n)}{\partial\alpha^2} \end{bmatrix}$$

4.2 Inverse Gaussian and gamma regression modeling on the percentage of stunting in Jepara

To determine the performance of inverse Gaussian and Gamma regression in a case, they will be applied to stunting data to obtain the causal factors. A distribution fit test is performed to make sure the stunting data has an inverted Gaussian and gamma distribution. The Variance Ratio test is used to test distributions. Alpha 0.05 was chosen because the majority of the data used came from social surveys which did not require high precision. The result of inverse Gaussian is a failed rejected null hypothesis because $F < F_{0.05;20;16}$ (2,276 < 13,271). The same result occurs in the Gamma distribution test which results in the acceptance of null hypothesis because $F < F_{0.05;20;16}$ (8,13 < 13,271). Therefore, the data has an Inverse Gaussian and Gamma.

The next stage is to detect multicollinear. To fulfill the assumption of no multicollinearity, the predictor variables must be independent of each other.

The next steps are parameter estimation and testing. MLE is applied to parameter estimation of inverse Gaussian and Gamma. The result of parameter estimation is obtained form Table 2. There are two parameter testing, simultaneous and partial parameter testing. G^2 of MLRT test statistics and Chi-square table for inverse Gaussian and Gamma regression can be seen from Table 1.

Table 1. Test statistics of MLRT

Parameter Regression	G^2	$\chi^2_{0,05;4}$
inverse Gaussian	33,49	9,49
Gamma	2,6	9,49

Table 1 indicates that all inverse Gaussian regression parameters are significant while the Gamma regression parameters are not significant simultaneously. This is because in the inverse Gaussian regression parameters $G^2 > \chi^2_{0,05;4}$ while $G^2 < \chi^2_{0,05;4}$ in Gamma regression. Based on these results, Gamma regression cannot be used to model the percentage of stunting in Jepara 2020. In addition, all Gamma regression parameters are not significant in the partial parameter test because $|Z| < Z_{0.05}$ as shown in Table 2.

Table 2. Estimator and partial parameter test

Inverse Gaussian regression			Gamma regression			
$\hat{\lambda}$	$\hat{\beta}$	|Z|	$\hat{\alpha}$	$\hat{\beta}$	|Z|	$Z_{0.05}$
	5.524e-05	4.384		5.833e-05	0.519	1.96
	-1.832e-03	-9.419		-2.108e-03	-1.276	1.96
	1.906e-04	3.022		2.828e-04	0.518	1.96
	8.021e-04	5.853		4.737e-04	0.394	1.96
2.315e-03		2.871	4.301e-03		0.072	1.96

Table 2 shows that in the inverse Gaussian regression model $|Z| > Z_{0.05}$ so that all parameters are partially significant. In contrast to that $|Z| < Z_{0.05}$ in the Gamma regression so they are not significant as explained in the simultaneous parameter test. Therefore, the best model to find factors that influence stunting in Jepara is inverse Gaussian regression. This result is strengthened by the Akaike Information Criterion (AIC) value. AIC is a criteria for selecting the best model based on the likelihood function. Inverse Gaussian regression has a smaller AIC than Gamma regression which can be seen in Table 3.

Table 3. AIC

Model	AIC
inverse Gaussian	124,58
Gamma	129,42

Based on Table 3, we can obtain AIC value of inverse Gaussian smaller than Gamma regression. Furthermore, it can be said that inverse Gaussian regression is the best model to determine the factors causing the percentage of stunting in Jepara. The best model on stunting percentage is

$$\hat{\mu}_i = \exp\left(0.1255 + 0.000055x_{i1} - 0.0018x_{i2} + 0.00019x_{i3} + 0.0008x_{i4}\right), \quad i = 1, 2, ..., n \tag{17}$$

Based on the results of the best model, it can be stated that exclusive breastfeeding, low birth weight babies, clean drinking water facilities, the number of posyandu have an influence on the percentage of stunting in Jepara.

5. CONCLUSION

Inverse Gaussian and gamma are two probability distributions that have the same s positive continuous random variables. However, when applied to stunting data in Jepara, it was found that the inverse Gaussian regression model was the best model. Exclusive breastfeeding, low birth weight babies, clean drinking water facilities, and the number of Integrated Service Post (posyandu) influence the percentage of stunting in Jepara by 84,3%.

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