JURNAL MATEMATIKA, STATISTIKA DAN KOMPUTASI e-ISSN: 2614-8811
Published by Departement of Mathematics, Hasanuddin University, Indonesia
https://journal.unhas.ac.id/index.php/jmsk/index
Vol. 21, No. 2, January 2025, pp. 538-543
DOI: 10.20956/j.v21i2.42165

On 2-Primal Quinary Semiring and its Characterizations by Special Subsets

Tuhfatul Janan

Institut Ahmad Dahlan Probolinggo
***Email**: tuhfatuljanan4@gmail.com*
Received: 22 November 2024, revised: 18 December 2024, accepted: 20 December 2024

Abstract

In this research, we introduce a new concept of 2-primal quinary semiring and its characterizations by utilizing special subsets. This concept is a generalization of 2-primal ternary semiring. The method of this research is a literature study on scientific articles in international journals. This research starts from concept of quinary semiring and some its ideals, then continues by studying the basic concept of 2-primal quinary semiring, including weakly and strongly nilpotent sets. Next, we define some special subsets of quinary semiring, and then provide some of their properties. Through this special subsets, we provide characterizations of 2-primal quinary semiring.
Keywords: prime ideal, insertion property, quinary semiring, 2-primal quinary semiring

1. INTRODUCTION

In 1932, Lehmer [11] proposed the notion of ternary semigroups theory, concentrating on regular and completely regular ternary semigroups. Additionally, he explored the standard embedding of ternary semigroups and showcased their applications. In another publication, Dutta and Kar [4] presented another algebraic structure known as a ternary semirings, along with regular ternary semirings and k-regular ternary semirings. They investigated various properties associated with them.

As it has progressed, many other authors have contributed to the study of ternary semirings, such as Kar and Shikari in [9] about soft ternary semirings, Kellil in [10] about strong ternary semirings, and Dutta and others in [3] about power ternary semirings. Subsequently, in separate studies documented in [6] and [1], are proposed the notion of k-regular additive ternary semiring and intra-regular ternary semirings, respectively. Furthermore, a series of works in [13], [14], [15], [17] and [12] identified the special ideals in ternary semirings such as k-hybrid ideals, a-ideals, tri-ideals, full k-ideals, and p-prime bi-ideal, respectively.

The notion of 2-primal rings was presented by Birkenmeier and others in [2] through left near rings. They defined a 2-primal ring if its radical set matches its nilpotent set. Paykan and Moussavi developed that research in [16] about some characterizations of 2-primal skew generalized power series rings. In 2015, Dutta and Mandal [5] introduced the notion of 2-primal ternary semiring, extending the findings of 2-primal rings to this context. Janan and Irawati [8] continued the research in [5] and explored some additional specialized subsets, aiming to derive alternative properties.

Furthermore, Janan [7] proposed the notion of quinary semiring and some properties of its special subsets. In this research, we present the notion of 2-primal quinary semiring and its properties by special subsets of quinary semiring. We introduce some definitions and properties of 2-primal quinary semiring. We also propose a generalized version of special subsets discussed in [5], denoted by $K(I), \overline{K(I)}, K_I$, and $\overline{K_I}$, then we explore some properties of them. Moreover, we will utilize these special subsets to characterize 2-primal quinary semiring.

2. METHOD

In this research, we use method of literature study on scientific articles in international journals. First, we give some definitions and properties of quinary semiring, a generalization of ternary semiring, and its ideals, such as prime ideal, fully semiprime ideal, and ideal which has the insertion property. Next, we introduce some concepts of nilpotent element and 2-primal quinary semiring. After that, we propose special subsets of quinary semiring, then we will characterize 2-primal quinary semiring by utilizing these special subsets.

3. RESULTS

3.1 Quinary Semiring and its Ideals

In this section, we give some definitions and properties of quinary semiring and its ideals, such as prime ideal, fully semiprime ideal, and ideal which has the insertion property.

Definition 3.1.1 *A quinary semiring is defined by a nonempty set Q, along with a binary addition and a quinary multiplication operations, forms an additive commutative semigroup and satisfies the succeeding criterias*:

(1) $(q_1q_2q_3q_4q_5)q_6q_7q_8q_9 = q_1(q_2q_3q_4q_5q_6)q_7q_8q_9 = q_1q_2(q_3q_4q_5q_6q_7)q_8q_9 = q_1q_2q_3(q_4q_5q_6q_7q_8)q_9 = q_1q_2q_3q_4(q_5q_6q_7q_8q_9)$,
(2) $(q_1 + q_2)q_3q_4q_5q_6 = q_1q_3q_4q_5q_6 + q_2q_3q_4q_5q_6$,
(3) $q_1(q_2 + q_3)q_4q_5q_6 = q_1q_2q_4q_5q_6 + q_1q_3q_4q_5q_6$,
(4) $q_1q_2(q_3 + q_4)q_5q_6 = q_1q_2q_3q_5q_6 + q_1q_2q_4q_5q_6$,
(5) $q_1q_2q_3(q_4 + q_5)q_6 = q_1q_2q_3q_4q_6 + q_1q_2q_3q_5q_6$,
(6) $q_1q_2q_3q_4(q_5 + q_6) = q_1q_2q_3q_4q_5 + q_1q_2q_3q_4q_6$

for any $q_1, q_2, q_3, q_4, q_5, q_6, q_7, q_8, q_9 \in Q$.

Example. Set of all imaginary number $\mathbb{I} = \{ai \mid a \in \mathbb{R}, i^2 = -1\}$.

Definition 3.1.2 *Let Q be a quinary semiring. Then an element $e \in Q$ is said to be an identity element if $xeeee = exeee = eexee = eeexe = eeeex = x$ for any $x \in Q$. If Q has an identity element then Q is said to be a quinary semiring with identity. In this paper, a quinary semiring Q refers to a quinary semiring Q with identity.*

Definition 3.1.3 *An additive subsemigroup I of a quinary semiring Q is said to be an ideal of Q if $xq_1q_2q_3q_4, q_1xq_2q_3q_4, q_1q_2xq_3q_4, q_1q_2q_3xq_4, q_1q_2q_3q_4x \in I$ for any $x \in I$ and $q_1, q_2, q_3, q_4 \in Q$.*

Definition 3.1.4 *An ideal of a quinary semiring Q is said to be generated by $q \in Q$ if $\langle q \rangle = QQQQqQQQQ$.*

Definition 3.1.5 *An ideal I of a quinary semiring Q is said to be a prime ideal of Q if $VWXYZ \subseteq I$ implies $V \subseteq I$ or $W \subseteq I$ or $X \subseteq I$ or $Y \subseteq I$ or $Z \subseteq I$ for any ideals V, W, X, Y, Z of Q. Moreover, I is said to be a fully semiprime ideal of Q if $x^5 \in I$ implies $x \in I$ for any $x \in Q$.*

Proposition 3.1.6 *Let Q be a quinary semiring and I be an ideal of Q. Then I is a prime ideal of Q if* *and only if $vQQQQwQQQQxQQQQyQQQQz \subseteq I$ satisfies $v \in I$ or $w \in I$ or $x \in I$ or $y \in I$ or $z \in I$ for any $v, w, x, y, z \in Q$.*

Proof. ($\Rightarrow$). Let $vQQQQwQQQQxQQQQyQQQQz \subseteq I$ for any $v, w, x, y, z \in Q$. Since Q is a quinary semiring with identity, then $\langle v\rangle = QQQQvQQQQ$. Therefore,

$$\begin{aligned}\langle v\rangle\langle w\rangle\langle x\rangle\langle y\rangle\langle z\rangle &= QQQQv(QQQQQ)QQQw(QQQQQ)QQQx(QQQQQ)QQQy(QQQQQ)QQQz\\ &\quad QQQQ\\ &\subseteq QQQQ(vQQQQwQQQQxQQQQyQQQQz)QQQQ\\ &\subseteq (QQQQI)QQQQ \subseteq IQQQQ \subseteq I\end{aligned}$$

Since I is a prime ideal of Q, then $\langle v\rangle \subseteq I$ or $\langle w\rangle \subseteq I$ or $\langle x\rangle \subseteq I$ or $\langle y\rangle \subseteq I$ or $\langle z\rangle \subseteq I$. As a result, we have $v \in I$ or $w \in I$ or $x \in I$ or $y \in I$ or $z \in I$.

($\Leftarrow$). Let $VWXYZ \subseteq I$ for any ideals V, W, X, Y, Z of Q. Suppose $W, X, Y, Z \nsubseteq I$. Then there exist $w \in W, x \in X, y \in Y,$ and $z \in Z$ such that $w, x, y, z \notin I$. Therefore, for any $v \in V$ we have $vQQQQwQQQQxQQQQyQQQQz \subseteq (VQQQQ)(WQQQQ)(XQQQQ)(YQQQQ)(ZQQQQ) \subseteq VWXYZ \subseteq I$. Hence, $v \in I$ which implies $V \subseteq I$. Thus, I is a prime ideal of Q.

Definition 3.1.7 *A nonempty subset T of a quinary semiring Q is said to be an t-system if for any $v, w, x, y, z \in T$ there exist $q_1, q_2, q_3, \dots, q_{16} \in Q$ such that $vq_1q_2q_3q_4wq_5q_6q_7q_8xq_9q_{10}q_{11}q_{12}yq_{13}q_{14}q_{15}q_{16}z \in T$.*

Proposition 3.1.8 *Let Q be a quinary semiring and I be an ideal of Q. Then I is a prime ideal of Q if and only if the complement of I, denoted by I^C is an t-system.*

Proof. ($\Rightarrow$). Suppose that I^C is not an t-system. Then there exist $v, w, x, y, z \in I^C$ such that for any $q_1, q_2, q_3, \dots, q_{16} \in Q$ implies $vq_1q_2q_3q_4wq_5q_6q_7q_8xq_9q_{10}q_{11}q_{12}yq_{13}q_{14}q_{15}q_{16}z \in I$. Therefore, $vQQQQwQQQQxQQQQyQQQQz \subseteq I$. Since I is a prime ideal of Q, by Proposition 3.1.6, we have $v \in I$ or $w \in I$ or $x \in I$ or $y \in I$ or $z \in I$. This is a contradiction. As a result, I^C is an t-system.

($\Leftarrow$). Let I^C is an t-system. Then for any $v, w, x, y, z \in I^C$ there exist $q_1, q_2, q_3, \dots, q_{16} \in Q$ such that $vq_1q_2q_3q_4wq_5q_6q_7q_8xq_9q_{10}q_{11}q_{12}yq_{13}q_{14}q_{15}q_{16}z \notin I$. Therefore, we have $vQQQQwQQQQxQQQQyQQQQz \nsubseteq I$. As a result, by Proposition 3.1.6, I is a prime ideal of Q.

Definition 3.1.9 *An ideal I of a quinary semiring Q has the insertion property if $vwxyz \in I$ implies $vQQQQwQQQQxQQQQyQQQQz \subseteq I$ for any $v, w, x, y, z \in Q$.*

3.2 Concept of 2-Primal Quinary Semiring

In this section, we introduce some concepts of nilpotent element and 2-primal quinary semiring.

Definition 3.2.1 *Let Q be a quinary semiring. Then an element $x \in Q$ is said to be a weakly nilpotent if there exists $n \in \mathbb{Z}^+$ which implies $x^{4n+1} = 0$. Moreover, the set of all weakly nilpotent elements of Q is denoted by $\mathcal{N}(Q)$.*

Proposition 3.2.2 *Let Q be a quinary semiring, $\mathcal{P}(Q)$ be the intersection of all prime ideals of Q. Then $\mathcal{P}(Q) \subseteq \mathcal{N}(Q)$.*

Proof. Let $x \notin \mathcal{N}(Q)$. Then for any $n \in \mathbb{Z}^+$ implies $x^{4n+1} \neq 0$. Define $T = \{x^{4n+1} \mid n \in \mathbb{Z}_0^+\}$. Then for any $x^{4n_1+1}, x^{4n_2+1}, x^{4n_3+1}, x^{4n_4+1}, x^{4n_5+1} \in T$ there exists $x \in Q$ such that

$$\begin{aligned}x^{4n_1+1}x^4x^{4n_2+1}x^4x^{4n_3+1}x^4x^{4n_4+1}x^4x^{4n_5+1} &= x^{4n_1+5}x^{4n_2+5}x^{4n_3+5}x^{4n_4+5}x^{4n_5+1}\\ &= x^{4n_1+4n_2+4n_3+4n_4+4n_5+5+5+5+5+1}\\ &= x^{4(n_1+n_2+n_3+n_4+n_5+5)+1} \in T.\end{aligned}$$

Therefore, T is an t-system not containing 0. By Proposition 3.1.8, T^C is a prime ideal of Q. Since $x =$

$x^{4.0+1} \in T$, then $x \notin T^C$. Hence, $x \notin \mathcal{P}(Q)$. As a result, we have $\mathcal{P}(Q) \subseteq \mathcal{N}(Q)$.

Proposition 3.2.3 *Let Q be a quinary semiring. Then $\mathcal{P}(Q)$ is an ideal of Q.*

Proof. By the definition of $\mathcal{P}(Q)$, clearly that $\mathcal{P}(Q) \subseteq Q$. Since $0 \in I$ for any prime ideal I of Q, then $0 \in \mathcal{P}(Q)$ which implies $\mathcal{P}(Q) \neq \emptyset$. Now, let $x_1, x_2 \in \mathcal{P}(Q)$. Then $x_1, x_2 \in I$ for any prime ideal I of Q. Therefore, $x_1 + x_2 \in I$ for any prime ideal I of Q. Hence, $x_1 + x_2 \in \mathcal{P}(Q)$. Now, let $x \in \mathcal{P}(Q)$ and $q_1, q_2, q_3, q_4 \in Q$. Therefore, $x \in I$ for any prime ideal I of Q. Hence, $q_1q_2q_3q_4x, q_1q_2q_3xq_4, q_1q_2xq_3q_4, q_1xq_2q_3q_4, xq_1q_2q_3q_4 \in I$ for any prime ideal I of Q. Thus, $q_1q_2q_3q_4x, q_1q_2q_3xq_4, q_1q_2xq_3q_4, q_1xq_2q_3q_4, xq_1q_2q_3q_4 \in \mathcal{P}(Q)$. As a result, $\mathcal{P}(Q)$ is an ideal of Q.

Definition 3.2.4 *Let Q be a quinary semiring. Then an element $x \in Q$ is said to be a strongly nilpotent if there exists $n \in \mathbb{Z}^+$ such that $(xQQQ)^n x = \{0\}$. Moreover, Q is said to be a strongly nilpotent if any $x \in \mathcal{N}(Q)$ is strongly nilpotent.*

Definition 3.2.5 *A quinary semiring Q is said to be a 2-primal quinary semiring if $\mathcal{P}(Q) = \mathcal{N}(Q)$.*

Lemma 3.2.6 *Let Q be a quinary semiring. Then Q is a 2-primal quinary semiring if and only if $\mathcal{P}(Q)$ is a fully semiprime ideal of Q.*

Proof. ($\Rightarrow$). Let $x^5 \in \mathcal{P}(Q)$ for any $x \in Q$. Since Q is a 2-primal quinary semiring, then $x^5 \in \mathcal{N}(Q)$. Therefore, there exists $n \in \mathbb{Z}^+$ such that $(x^5)^{(4n+1)} = 0$. Hence, $x^{4(5n+1)+1} = x^{20n+5} = (x^5)^{(4n+1)} = 0$. Since $5n + 1 \in \mathbb{Z}^+$, then $x \in \mathcal{N}(Q) = \mathcal{P}(Q)$. As a result, $\mathcal{P}(Q)$ is a fully semiprime ideal of Q.

($\Leftarrow$). By Proposition 3.2.2, $\mathcal{P}(Q) \subseteq \mathcal{N}(Q)$. Now, let $x \in \mathcal{N}(Q)$. Then there exists $n \in \mathbb{Z}^+$ such that $x^{4n+1} = 0 \in \mathcal{P}(Q)$. Since $\mathcal{P}(Q)$ is a fully semiprime ideal of Q, then $x \in \mathcal{P}(Q)$. Hence, $\mathcal{N}(Q) \subseteq \mathcal{P}(Q)$ which implies $\mathcal{P}(Q) = \mathcal{N}(Q)$. As a result, Q is a 2-primal quinary semiring.

3.3 Characterizations of 2-Primal Quinary Semiring

In this section, we propose a generalized version of special subsets discussed in [5], then we characterize 2-primal quinary semiring by utilizing these special subsets.

Definition 3.3.1 *Let Q be a quinary semiring and I be a prime ideal of Q. Then we define*

$$\underline{K(I)} = \{x \in Q \mid xQQQyQQQ \subseteq \mathcal{P}(Q) \text{ for some } y \in I^C\},$$
$$\overline{K(I)} = \{x \in Q \mid (xQQQ)^n x \subseteq K(I) \text{ for some } n \in \mathbb{Z}^+\},$$
$$\underline{K_I} = \{x \in Q \mid xyQQQ \subseteq \mathcal{P}(Q) \text{ for some } y \in I^C\},$$
$$\overline{K_I} = \{x \in Q \mid (xQQQ)^n x \subseteq K_I \text{ for some } n \in \mathbb{Z}^+\}.$$

Proposition 3.3.2 *Let Q be a quinary semiring and I be a prime ideal of Q. Then*

(1) $K(I) \subseteq I$,
(2) $K(I) \subseteq \overline{K(I)} \subseteq \overline{K_I}$,
(3) $K(I) \subseteq K_I \subseteq \overline{K_I}$.

Proof. It is clearly by using Definition 3.3.1, Proposition 3.1.6, and Proposition 3.2.3.

Theorem 3.3.3 *Let Q be a strongly nilpotent quinary semiring. Then the succeeding properties are equivalent:*

(1) *Q is a 2-primal quinary semiring,*
(2) *$\mathcal{P}(Q)$ has the insertion property,*
(3) *$K(I)$ has the insertion property for any prime ideal I of Q,*

(4) $K(I) = \overline{K(I)} = K_I = \overline{K_I}$ *for any prime ideal* I *of* Q.

Proof. (1) ⇒ (2). Let $vwxyz \in \mathcal{P}(Q)$ for any $v, w, x, y, z \in Q$. By Proposition 3.2.3, $(wxyzv)^5 = wxyz(vwxyz)(vwxyz)(vwxyz)(vwxyz)v \in \mathcal{P}(Q)$. Since Q is a 2-primal quinary semiring, by Lemma 3.2.6, $wxyzv \in \mathcal{P}(Q)$. Therefore, by Proposition 3.2.3 we have $(xyzvQQQQw)^5 = xyzvQQQQ(wxyzv)QQQQ(wxyzv)QQQQ(wxyzv)QQQQ(wxyzv)QQQQw \subseteq \mathcal{P}(Q)$. Since Q is a 2-primal quinary semiring, by Lemma 3.2.6, $xyzvQQQQw \subseteq \mathcal{P}(Q)$. Hence, by Proposition 3.2.3 we have $(yzvQQQQwQQQQx)^5 = yzvQQQQwQQQQ(xyzvQQQQw)QQQQ(xyzvQQQQw)QQQQ(xyzvQQQQw)QQQQ(xyzvQQQQw)QQQQx \subseteq \mathcal{P}(Q)$. Since Q is a 2-primal quinary semiring, by Lemma 3.2.6, we have $yzvQQQQwQQQQx \subseteq \mathcal{P}(Q)$. Similarly, $(zvQQQQwQQQQxQQQQy)^5 \subseteq \mathcal{P}(Q)$ by Proposition 3.2.3 and implies $zvQQQQwQQQQxQQQQy \subseteq \mathcal{P}(Q)$ by Lemma 3.2.6 Moreover, $(vQQQQwQQQQxQQQQyQQQQz)^5 \subseteq \mathcal{P}(Q)$ by Proposition 3.2.3 and implies $vQQQQwQQQQxQQQQyQQQQz \subseteq \mathcal{P}(Q)$ by Lemma 3.2.6. As a result, $\mathcal{P}(Q)$ has the insertion property.

(2) ⇒ (3). Let $vwxyz \in K(I)$ for any $v, w, x, y, z \in Q$ and prime ideal I of Q. Then there exists $b \in I^C$ which implies $vwxyzQQQQbQQQ \subseteq \mathcal{P}(Q)$. Since $\mathcal{P}(Q)$ has the insertion property, then $vQQQQwQQQQxQQQQyQQQQzQQQQbQQQ \subseteq \mathcal{P}(Q)$. Hence, $vQQQQwQQQQxQQQQyQQQQz \subseteq K(I)$. As a result, $K(I)$ has the insertion property for any prime ideal I of Q.

(3) ⇒ (1). By Proposition 3.2.2, $\mathcal{P}(Q) \subseteq \mathcal{N}(Q)$. Suppose that there exists $x \in \mathcal{N}(Q)$ but $x \notin \mathcal{P}(Q)$. Then there exists prime ideal I of Q which implies $x \in I^C$. By Proposition 3.1.8, I^C is an t-system. Therefore, there exist $q_1, q_2, q_3, \dots, q_{16} \in Q$ such that $xq_1q_2q_3q_4xq_5q_6q_7q_8xq_9q_{10}q_{11}q_{12}xq_{13}q_{14}q_{15}q_{16}x \in I^C$. By repeating this argument, we have $(xQQQQ)^{4n}x \nsubseteq I$ for some $n \in \mathbb{Z}^+$. Next, since $x \in \mathcal{N}(Q)$, then there exists $n \in \mathbb{Z}^+$ which implies $x^{4n+1} = 0 \in K(I)$. Since $K(I)$ has the insertion property for any prime ideal I of Q, then $(xQQQQ)^{4n}x \subseteq K(I)$. By Proposition 3.3.2, we have $(xQQQQ)^{4n}x \subseteq I$. This is a contradiction. Hence, $\mathcal{N}(Q) \subseteq \mathcal{P}(Q)$ which implies $\mathcal{P}(Q) = \mathcal{N}(Q)$. As a result, Q is a 2-primal quinary semiring.

(1) ⇒ (4). By Proposition 3.3.2, we have $K(I) \subseteq \overline{K(I)} \subseteq \overline{K_I}$ and $K(I) \subseteq K_I \subseteq \overline{K_I}$ for any prime ideal I of Q. Now, let $x \in \overline{K_I}$. Then there exists $n \in \mathbb{Z}^+$ such that $(xQQQ)^n x \subseteq K_I$. Therefore, there exists $y \in I^C$ such that $(xQQQ)^n xyQQQ \subseteq \mathcal{P}(Q)$. Note that $x^{4n+1}yQQQ = (xxxx)^n xyQQQ \subseteq (xQQQ)^n xyQQQ \subseteq \mathcal{P}(Q)$. Since Q is a 2-primal quinary semiring, by (2) we have $(xQQQQQQQQ)^{4n}(xQQQQyQQQ)QQQQQQQQQQQQ \subseteq \mathcal{P}(Q)$. Since Q has an identity element e, $(xQQQQyQQQ)^{4n+1} \subseteq \mathcal{P}(Q)$. Since Q is a 2-primal quinary semiring, by Lemma 3.2.6, $xQQQQyQQQ \subseteq \mathcal{P}(Q)$ which implies $x \in K(I)$. Thus, $\overline{K_I} \subseteq K(I)$. As a result, $K(I) = \overline{K(I)} = K_I = \overline{K_I}$ for any prime ideal I of Q.

(4) ⇒ (1). By Proposition 3.2.2, $\mathcal{P}(Q) \subseteq \mathcal{N}(Q)$. Suppose that there exists $x \in \mathcal{N}(Q)$ but $x \notin \mathcal{P}(Q)$. Then there exists prime ideal I of Q which implies $x \notin I$. Now, since Q is a strongly nilpotent quinary semiring, then there exists $n \in \mathbb{Z}^+$ which implies $(xQQQ)^n x = \{0\} \subseteq K_I$. Therefore, $x \in \overline{K_I}$. Since $K(I) = \overline{K(I)} = K_I = \overline{K_I}$, we have $x \in K(I)$. By Proposition 3.3.2, $x \in I$. This is a contradiction. Hence, $\mathcal{N}(Q) \subseteq \mathcal{P}(Q)$ which implies $\mathcal{P}(Q) = \mathcal{N}(Q)$. As a result, Q is a 2-primal quinary semiring.

4. CONCLUSION

A quinary semiring Q is a 2-primal quinary semiring if and only if $\mathcal{P}(Q)$ is a fully semiprime ideal of Q. By utilizing special subsets, some characterizations of 2-primal quinary semiring Q are $\mathcal{P}(Q)$ and $K(I)$ has the insertion property, and $K(I) = \overline{K(I)} = K_I = \overline{K_I}$ for any prime ideal I of Q.

REFERENCES

[1] Bashir, S., Ali Al-Shamiri, M. M., Khalid, S., & Mazhar, R., 2023. Regular and Intra-Regular

Ternary Semirings in Terms of m-Polar Fuzzy Ideals. *Symmetry (Basel).*, Vol. 15, No. 3, 591. https://doi.org/10.3390/sym15030591

[2] Birkenmeier, G. F., Heatherly, H. E., & Lee, E. K., 1992. Completely prime ideals and associated radicals. *Proc. Bienn. Ohio State-Denison Conf.*, 102–129. https://doi.org/10.1142/9789814535816

[3] Dutta, T. K., Kar, S., & Das, K., 2015. Power ternary semirings. *Afrika Mat.*, Vol. 26, No. 7, 1483–1494. https://doi.org/10.1007/s13370-014-0300-9

[4] Dutta, T. K., & Kar, S., 2003. On Regular Ternary Semirings. *Adv. Algebr.*, 343–355. https://doi.org/10.1142/9789812705808_0027

[5] Dutta, T. K., & Mandal, S., 2015. Some Characterizations of 2-primal Ternary Semiring. *Southeast Asian Bull. Math.*, Vol. 39, No. 6, 769–783. http://www.seams-bull-math.ynu.edu.cn/archive.jsp

[6] Ingale, K. J., Bendale, H. P., Bonde, D. R., & Chaudhari, J. N., 2022. ON K-REGULAR ADDITIVE TERNARY SEMIRINGS. *J. Indian Math. Soc.*, Vol. 89, No. 1–2, 72–83. https://doi.org/10.18311/jims/2022/29309

[7] Janan, T., 2024. On quinary semiring and properties of its special subsets. *Jurnal Matematika, Statistika dan Komputasi*, Vol. 21, No. 1, 78-87. https://doi.org/10.20956/j.v21i1.35615

[8] Janan, T., & Irawati, 2023. Characterizations of 2-Primal Ternary Semiring using Special Subsets of Ternary Semiring. *Limits J. Math. Its Appl.*, Vol. 20, No. 1, 97–105. http://dx.doi.org/10.12962/limits.v20i1.12965

[9] Kar, S., & Shikari, A., 2016. Soft ternary semirings. *Fuzzy Inf. Eng.*, Vol. 8, No. 1, 1–15. https://doi.org/10.1016/j.fiae.2016.03.001

[10] Kellil, R., 2016. Idempotent and Inverse Elements in Strong Ternary Semirings. in *International Mathematical Forum*, Vol. 11, No. 5, 201–211. http://dx.doi.org/10.12988/imf.2016.51191

[11] Lehmer, D. H., 1932. A ternary analogue of abelian groups. *Am. J. Math.*, Vol. 54, No. 2, 329–338. https://doi.org/10.2307/2370997

[12] Manivasan, S., & Parvathi, S., 2022. The dissertation on P-prime bi-ideal in ternary semirings. *AIP Conf. Proc.*, Vol. 2516, No. 1. https://doi.org/10.1063/5.0108621

[13] Muhiuddin, G., Catherine Grace John, J., Elavarasan, B., Porselvi, K., & Al-Kadi, D., 2022. Properties of k-hybrid ideals in ternary semiring. *J. Intell. Fuzzy Syst.*, Vol. 42, No. 6, 5799–5807. https://doi.org/10.3233/JIFS-212311

[14] Palanikumar, M., & Arulmozhi, K., 2021a. New approach towards A-ideals in Ternary Semirings. *Ann. Commun. Math.*, Vol. 4, No. 2, 114. https://doi.org/10.62072/acm.2021.040203

[15] Palanikumar, M., & Arulmozhi, K., 2021b. On various tri-ideals in ternary semirings. *Bull. Int. Math. Virtual Inst.*, Vol. 11, No. 1, 79–90. https://doi.org/10.7251/BIMVI2101079P

[16] Paykan, K., & Moussavi, A., 2020. Some characterizations of 2-primal skew generalized power series rings. *Commun. Algebr.*, Vol. 48, No. 6, 2346–2357. https://doi.org/10.1080/00927872.2020.1713326

[17] Sunitha, T., Reddy, U. N., & Shobhalatha, G., 2021. A note on full k-ideals in ternary semirings. *Indian J. Sci. Technol.*, Vol. 14, No. 21, 1786–1790. https://doi.org/10.17485/IJST/v14i21.150