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# Estimating Reinsurance Premiums Using Pareto Conjugate Priors and Extreme Value Methods: Studies Case of Fire Insurance Claims in Denmark

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**Abstract**
Loss distributions in insurance are typically right-skewed with heavy tails. As a result, modelling such distributions often involves the use of heavy-tailed distributions, such as the Pareto family, Cauchy, Student-t, and mixture distributions. This study employs the Generalized Inverse Gaussian (GIG) distribution as a conjugate prior to the Pareto distribution. The GIG distribution is characterized by three parameters and includes the modified Bessel function of the third kind in its density, which makes parameter estimation using the likelihood method challenging. Therefore, a Bayesian estimation approach is adopted, utilizing two prior distributions from the GIG family: the Inverse Gaussian and the Reciprocal Inverse Gaussian. The modelling is carried out within the framework of Extreme Value Theory (EVT), focusing on excess values over a specified threshold and the probability of claims exceeding that threshold. The results obtained from this analysis can be used to derive a premium estimation formula that insurance companies can apply when reinsuring their claims with a reinsurance company.

**Keywords:** Loss Distribution, Bayesian Estimation, Premium Reinsurance, Extreme Value Theory

## 1. INTRODUCTION

The primary role of an insurance company is to assume risk. This responsibility arises from the uncertainty surrounding both the timing and magnitude of claims, as policyholders are financially protected within a defined time frame and for a specific sum [1]. As the global insurance industry expands and the number of policyholders increases, the volume of insurance policies also rises. Consequently, the company faces a higher risk of having to pay substantial amounts when claims are made. One effective risk management strategy is transferring part of the risk to a third party, known as reinsurance. Reinsurance, often described as "insurance for insurers" [14]. serves as a form of insurance and also functions as a crucial risk management tool in insurance companies due to its ability to provide coverage for major financial losses [13]. Reinsurance serves as a financial instrument that allows insurance companies to redistribute a portion of their risk exposures and liabilities to reinsurance entities, with the objective of reducing concentration risk and enhancing overall risk management strategies[15].

Over time, insurance companies accumulate historical data on claims they have paid. This data forms what is known as the loss distribution. In insurance, loss distributions are typically heavy-tailed, characterized by a thick right tail. One effective method for modeling rare but high-impact events is Extreme Value Theory (EVT). EVT offers a robust framework for analyzing such extreme events [8]. Within EVT, there are two main modeling approaches: the first involves modeling the distribution of maxima, commonly referred to as the Generalized Extreme Value (GEV) distribution; the second focuses on modeling excesses over a specified threshold, often used to capture tail behavior more precisely [7].

Several heavy-tailed data distributions commonly encountered in risk modeling include the Pareto family, Cauchy, Student-t, and various mixture distributions. In this study, the Pareto conjugate prior is employed, specifically the Generalized Inverse Gaussian (GIG) distribution, which is characterized by three parameters and incorporates the modified Bessel function of the third kind [18]. Since parameter estimation for the GIG distribution cannot be performed using the traditional likelihood approach, Bayesian estimation is applied instead, utilizing two prior distributions from the GIG family: the Inverse Gaussian and the Reciprocal Inverse Gaussian. The results obtained from this Bayesian estimation are then used to derive an exact credibility formula, which enables the calculation of estimated premiums that an insurance company must pay when reinsuring its claims with a reinsurance provider [3].

This research contributes to the literature by introducing the use of Generalized Inverse Gaussian (GIG) conjugate priors, specifically the Inverse Gaussian and Reciprocal Inverse Gaussian for modeling the severity claim of extreme fire insurance claim in Denmark. The main contribution of this study lies in combining these conjugate priors with extreme value theory framework and Bayesian estimation, resulting in a flexible and robust approach to reinsurance premium calculation. This approach also captures uncertainty in the tail of distribution using vague priors.

## 2. LITERATURE REVIEW

### 2.1 Extreme Value Theory

Extreme Value Theory (EVT) is a well-established approach for modeling data with heavy-tailed distributions. It has been widely applied in various fields, including reliability engineering, insurance, hydrology, and climatology, particularly for forecasting the occurrence of rare but significant extreme events [9]. EVT comprises two primary modeling techniques. The first is maxima modeling, in which extreme values are defined as the maximum observations within a sample. These values asymptotically follow one of three extreme value distributions: Fréchet, Gumbel, or Weibull, which together form the basis of the Generalized Extreme Value (GEV) distribution. The second approach is threshold exceedance modeling, In classical statistical theory, the threshold for excess is treated as a fixed value $u \in \mathbb{R}$. In this context, only the subset of observations exceeding this threshold is utilized to model the conditional excess distribution [6]. When extreme events are characterized as values exceeding a specified threshold. The distribution of the excesses over this threshold converges asymptotically to the Generalized Pareto Distribution (GPD) [5].

### 2.2 Generalized Pareto Distribution Modeling

According to [5], this modeling approach focuses on observations that are sufficiently large to exceed a specified high threshold. The threshold represents a boundary value, beyond which the random variable $X$ ais considered to exhibit extreme behavior, whereas under normal conditions, its values are expected to lie below this limit. In the context of the Generalized Pareto Distribution (GPD), the threshold commonly denoted by $u$ also serves as the location parameter of the distribution. Observations that surpass this threshold are classified as extreme values, and the excess value is defined as the difference between an extreme value and the threshold itself. Let $X_i = \{X_1, X_2, \dots, X_n\}$ be a random sample of claim values exceeding the threshold $u$. The excess value are defined as

$$Y_i = X_i - u, X_i > u \quad (2.1)$$

The probability distribution of the excess value provided that $X_i > u$ is defined

$$F_T(y_i) = \frac{F(x_i + u) - F(u)}{1 - F(u)} \quad (2.2)$$

For $X_i = Y_i + u, X_i > u$, is obtained

$$F(x_i) = F_t(y_i)(1 - F(u)) + F(u) \quad (2.3)$$

## 2.3 Bayesian Estimation

In the classical approximation method, the $\theta$ parameter is an unknown fixed quantity. Sample $X_1, X_2, \dots, X_n$ were taken from the $\theta$-indexed population and based on the observed values in the sample, information about the $\theta$ was obtained. In the Bayesian approach, $\theta$ is seen as a quantity whose variance is described by a probability distribution (called the prior distribution). It is a subjective distribution based on a researcher's belief and is formulated before the data is collected. Then the sample is taken from the $\theta$-indexed population and the prior distribution is adjusted according to this sample information. The adjusted prior is called posterior distribution. This adjustment is made according to Bayes' rules, this is the reason why it is called Bayesian statistics.

**Definition 2.1.** *Let $\theta$ be the shape parameter of the Pareto distribution and $X$ denoted the observed excess values over threshold $u$. Bayes' theorem in probability is formulated in the form* [17]

$$\pi(\theta \mid x) = \frac{\pi(\theta) f(x \mid \theta)}{\int_{\Omega} \pi(\theta) f(x \mid \theta) d\theta} \quad (2.4)$$

This formulation reflects the posterior distribution of $\theta$ $(\pi(\theta|x))$ given the excess data where $X$the prior $(\pi(\theta))$ is updated through the likelihood function $(f(x|\theta))$. Prior distribution $\pi(\theta)$ is a subjective distribution based on the belief of a person and formulated before the data is taken.

**Definition 2.2.** *Let $X|\theta \sim f(x|\theta)$ represent the likelihood function and assume that the prior distribution of unknown parameter $\theta \in \Omega$ is $\pi(\theta)$. The Bayesian model specification can be summarized as* [12]

$$\begin{aligned} X \mid \theta &\sim f(x \mid \theta) \\ \theta &\sim \pi(\theta) \end{aligned} \quad (2.5)$$

This formulation describes the Bayesian hierarchical model, where $\pi(\theta)$ encodes prior beliefs about the parameter $\theta$, and the conditional distribution $f(x \mid \theta)$ represents the data-generating process. The sample distribution combined with the prior distribution will produce a posterior distribution. The posterior distribution expresses a person's belief about the parameter after the sample is taken.

**Definition 2.3.** *Distribution of $\theta$ if known the data is called posterior distribution* [12]

$$\pi(\theta \mid x) = \frac{f(x,\theta)}{f(x)} = \frac{f(x \mid \theta)\pi(\theta)}{f(x)} \quad (2.6)$$

*with $\boldsymbol{f}(\boldsymbol{x})$ is marginal distribution from $\boldsymbol{x}$ that can be calculated by*

$$f(x) = \begin{cases} \sum_{\theta} f(x \mid \theta)\pi(\theta), \theta & discrete \\ \int f(x \mid \theta)\pi(\theta), \theta & continue \end{cases} \quad (2.7)$$

Predictive distribution can show how observations in the next period are based on the information contained in the data. Based on the predictive distribution, the expectations of the predictive distribution can be determined, which will be very influential when determining the premium.

**Definition 2.4.** *Predictive distribution is a conditional distribution of the new observation if given past data. Predictive distribution can be calculated by* [12]

$$f_{Y|X}(y \mid x) = \int f_{Y|\Theta}(y \mid \theta)\pi_{\Theta|X}(\theta \mid x)\, d\theta \quad (2.8)$$

# 3. MATERIAL AND METHOD

## 3.1 Data

The dataset was sourced from the CASdatasets R package, represent high value property losses collected between by Reinsurance Copenhagen [2]. Each record in the dataset corresponds to an individual fire insurance claim exceeding one million Danish Krone (DKK), and all values have been adjusted for inflation to ensure consistency over time.

The dataset consists of 2,167 observations and is often used as a benchmark in actuarial studies, especially in modeling large claims due to its heavy-tailed nature. The claims are expressed in millions of DKK, with values ranging from 1.00 to 263.25, and the total sum of claims reaching 7,335.485 million DKK. This wide range and extreme variability make the dataset ideal for testing reinsurance pricing models under Extreme Value Theory (EVT).

Furthermore, the dataset exhibits strong leptokurtic characteristics, as indicated by a kurtosis value significantly higher than that of a normal distribution. This supports the use of heavy-tailed models such as the Generalized Pareto Distribution (GPD) or compound models in premium estimation. The credibility of the dataset is reinforced by its consistent appearance in scholarly literature focused on large claim analysis and reinsurance.

## 3.2 Assumptions

Suppose random variable $N$ and $X$ are defined respectively as the claim number and claim severity. Threshold is fixed at $u > 0$ and assume that

$$p^u = P(X > u) > 0 \quad (3.1)$$

$p^u$ is referred to as the tail probability, which represents the probability that a claim exceeds a specified threshold [18]. A new random variable, $N^u$ is then defined to represent the excess amount above the threshold that is, the conditional excess $u\ (X - u|X > u)$. Assuming the excess values are sufficiently large, their distribution can be approximated by a Generalized Pareto Distribution (GPD). Consequently, if the severity of claims is assumed to follow a Pareto-type distribution, the resulting model corresponds to a Pareto case. The total normalized excesses above the threshold can thus be modeled as a compound random variable.

$$S^u = \sum_{i=1}^{N^u} Y_i \quad (3.2)$$

$S^u = 0$ when $N^u = 0$ with $(Y_i)_{i=1}^{\infty}$ are excesses that have been normalized.

$$Y = \frac{X-u}{u}, X > u \quad (3.3)$$

Normalizing the excess amounts helps to avoid the need for estimating the location and scale parameters of the excess distribution. It is then assumed that there exists a continuous random variable $\Lambda^u > 0$ which is independent of time. This variable has a distribution function $W(\lambda) = P(\Lambda^u \le \lambda)$ and density function $w(\lambda)$, Under this assumption, the number of excess claims observed across different years is conditionally independent given $\Lambda^u$. In this context, $n$ denotes the total number of years observation, $n_i$ is the numer of excess claims observed in year $i$, and $N_i^u$ represents the total number of claims in year $i$ that exceed threshold $u$.

$$P\left(N_i^u = n_1, \ldots, N_k^u \mid \Lambda^u\right) = \prod_{i=1}^{k} P\left(N_i^u = n_i \mid \Lambda^u\right), \forall k \in \mathrm{N}, \forall n \in \mathrm{N}_0 \quad (3.4)$$

The conditional Poisson distribution $\Lambda^u$is

$$P\left(N_i^u = n \mid \Lambda^u\right) = \frac{\left(\Lambda^u\right)^n}{n!} e^{-\Lambda^u}, \forall i \in \mathrm{N}, \forall n \in \mathrm{N}_0 \quad (3.5)$$

Therefore, the unconditional distribution of the number of excess claims follows a mixed Poisson distribution, where the mixing occurs over the parameter $\lambda$ defined as follows

$$P\left(N_i^u = n\right) = \int_0^{\infty} \frac{\lambda^n}{n!} e^{-\lambda} w(\lambda)\, d\lambda, \forall i \in \mathrm{N}, \forall z_i \in \mathbf{R} \quad (3.6)$$

It is also assumed that the Pareto shape parameter is an absolute continuous random variable $Y > 0$, independent of time and independent of random parameter variable $\Lambda^u$. Then the Pareto index will be reciprocal $\Theta = 1/y > 0$. The random variable $\Theta$ should have cumulative distribution according to $\Pi(\theta) = P(\Theta = \theta)$, and its density function is $\pi(\theta)$. Random variable $(Y_i)_{i \in \mathbb{N}}$ is a conditional independent random variable $\Theta$, it means

$$P\left(Y_i \le y_i \mid \Theta\right) = \prod_{i=0}^{n} P\left(Y_i \le y_i \mid \Theta\right), \forall i \in \mathrm{N}, \forall z_i \in \mathbf{R} \quad (3.7)$$

The conditional Pareto distribution $\Theta$ is

$$P\left(Y_i \le y_i \mid \Theta\right) = 1 - \frac{1}{(1+y)^{\Theta}}, \forall i \in \mathrm{N}, \forall y > 0 \quad (3.8)$$

Under these assumptions, the unconditional normalized excess density function is mixed Pareto, that is

$$g(y) = \int_0^{\infty} \frac{\theta}{(1+y)^{\theta+1}} \pi(\theta)\, d\theta \quad (3.9)$$

Finally, will be assumed the independence of $(N_1^u, \ldots, N_k^u, \Lambda^u)$ and $(Y_1, \ldots, Y_n, \Theta)$ for every $k, n \in \mathbb{N}$. With the assumption $\Lambda^u$ and $\Theta$ independent, then the prior density joint can be written as a product

$$v(\lambda, \theta) = w(\lambda)\pi(\theta) \quad (3.10)$$

From this condition, the number of excess and a sample of normalized excess that have been observed during $k$ years. Let $(y_1, \ldots, y_n)$ denote the vector of normalized excess values observed across all $k$ years.

$$n = \sum_{i=1}^{k} n_i, (y_1, ..., y_n) \quad (3.11)$$

The joint posterior density for $(\Lambda^u, \Theta)$-vector random is

$$v(\lambda, \theta \mid n_1, ..., n_k, y_1, ..., y_n) = w(\lambda \mid n_1, ..., n_k)\pi(\theta \mid y_1, ..., y_n) \quad (3.12)$$

where

$$w(\lambda \mid n_1, ..., n_k) \propto \frac{\lambda^{\sum_{i=1}^{k} n_i}}{\prod_{i=1}^{k} n_i!} e^{-k\lambda} w(\lambda) \quad (3.13)$$

$$\pi(\theta \mid y_1, ..., y_n) \propto h(y_1, ..., y_n \mid \Theta = \theta)\pi(\theta)$$

and $h(y_1, \dots, y_n | \Theta = \theta)$ is likelihood function for normalized Pareto.

When the square root loss function is used, it is known that the Bayes estimates for both parameters correspond to their posterior means

$$\lambda(n_1, ..., n_k) = \int_0^\infty \lambda w(\lambda \mid n_1, ..., n_k) d\lambda \quad (3.14)$$

$$\theta(y_1, ..., y_n) = \int_0^\infty \theta\pi(\theta \mid y_1, ..., y_n) d\theta \quad (3.15)$$

And then the formula for calculating the reinsurance premium $(P)$ can be derived to replace the point estimate into each conditional distribution function, the mean of the conditional compound random variable $\left(S^u | \Lambda^u = \hat{\lambda}, \Theta = \hat{\theta}\right)$ can be written as follows

$$P(n_1, ..., n_k, y_1, ..., y_n) = \frac{\lambda(n_1, ..., n_k)}{\theta(y_1, ..., y_n)} p^u \quad (3.16)$$

for $\theta(y_1, \dots, y_n) > 0$

For threshold value $(u)$, equation (3.16) is

$$P(n_1, ..., n_k, y_1, ..., y_n) = \frac{\lambda(n_1, ..., n_k)u}{\theta(y_1, ..., y_n) - 1} p^u \quad (3.17)$$

### 3.3 Model Formulation

The prior conjugate Pareto distribution employed in this study is the Generalized Inverse Gaussian (GIG) distribution. The GIG family is a well-established probabilistic model [11]. Originally introduced by Étienne Halphen, the distribution was later popularized by Ole Barndorff-Nielsen. It has been applied in various contexts, including by [16] to construct a mixed Poisson distribution, and the latest by [4] which discusses the prior distribution suitable for studying dependence in reinsurance models. The GIG distribution is defined by three parameters $(\beta, \chi, \psi)$ and has the following density function

$$\pi_{\beta,\chi,\psi}(\theta) = \frac{\left(\psi/\chi\right)^{\frac{\beta}{2}}}{2K_\beta\left(\sqrt{\chi\psi}\right)} \theta^{\beta-1} e^{-\frac{1}{2}\left(\frac{\chi}{\theta} + \psi\theta\right)}, \theta > 0, \beta \in \mathbf{R}, \chi, \psi > 0 \quad (3.18)$$

where $K_\beta$ is Bessel function third type with $\beta$-indexed. The central moment and variance of GIG are

$$E\left(\Theta^{j}\right)=\left(\frac{\chi}{\psi}\right)^{\frac{j}{2}}\frac{K_{\beta+j}\left(\sqrt{\chi\psi}\right)}{K_{\beta}\left(\sqrt{\chi\psi}\right)} \quad (3.19)$$

$$Var\left(\Theta\right)=\frac{\chi}{\psi}\left[\frac{K_{\beta+2}\left(\sqrt{\chi\psi}\right)}{K_{\beta}\left(\sqrt{\chi\psi}\right)}-\frac{K_{\beta+1}^{2}\left(\sqrt{\chi\psi}\right)}{K_{\beta}^{2}\left(\sqrt{\chi\psi}\right)}\right] \quad (3.20)$$

In Equation (3.19), the index $j$ denotes the order of the moment of the random variable $\Theta$, with $E(\Theta_{\mathrm{j}})$ representing the $j$-th raw moment. This allows for the calculation of expectations such as the mean (for $j = 1$ yields the first moment) or higher-order moments when necessary. The posterior distribution is still GIG distributed with parameters

$$\begin{aligned} \beta' &= n+\beta \\ \chi' &= \chi \\ \psi' &= 2\sum_{i=1}^{n}\log\left(y_i+1\right)+\psi \end{aligned} \quad (3.21)$$

When $\chi, \psi > 0$, the predictive distribution can be obtained by substituting $\pi(\theta)$ as GIG density into (3.22)

$$g_{\beta,\chi,\psi}\left(y\right)=\sqrt{\frac{\chi}{\psi}}\frac{K_{\beta+1}\left(\sqrt{\chi\psi\left[1+\frac{2}{\psi}\log\left(1+y\right)\right]^{\frac{1}{2}}}\right)}{K_{\beta}\left(\chi\psi\right)\left(1+y\right)\left(1+\frac{2}{\psi}\log\left(1+y\right)\right)^{\left(\frac{\beta+1}{2}\right)}} \quad (3.22)$$

This predictive distribution is not easy to use for handling variable $y$ that appears in Bessel This predictive distribution is difficult to work with due to the presence of the variable y within the argument of the Bessel function. As a result, the prior distributions selected for use in this study are specific members of the GIG family namely, the Inverse Gaussian and the Reciprocal Inverse Gaussian distributions.

**Reciprocal gamma distribution** with parameters $\beta < 0, \chi > 0, \psi = 0$ has density function

$$\pi_{\beta,\chi,0}\left(\theta\right)=\left(\frac{2}{\chi}\right)^{\beta}\Gamma\left(-\beta\right)^{-1} \quad (3.23)$$

The predictive distribution obtained is

$$g_{\beta,\chi,0}\left(y\right)=\left(\frac{2}{\log\left(1+y\right)}\right)^{\frac{\beta+1}{2}}\frac{\chi^{\frac{1-\beta}{2}}}{\Gamma\left(-\beta\right)\left(1+y\right)}K_{\beta+1}\left(\sqrt{2\chi\log\left(1+y\right)}\right) \quad (3.24)$$

The central moment and variance of reciprocal gamma distribution are

$$E\left(\Theta^{j}\right)=\left(\frac{\chi}{\psi}\right)^{\frac{j}{2}}\frac{\Gamma\left(-\beta-k\right)}{\Gamma\left(-\beta\right)},\ j<-\beta \quad (3.25)$$

$$Var\left(\Theta\right)=-\left(\frac{\chi}{\psi}\right)^{2}\frac{1}{\left(\beta+1\right)^{2}\left(\beta+2\right)} \quad (3.26)$$

In Equation (3.25), the exponent $\frac{j}{2}$ corresponds to the order $j$ of the raw moment $E(\Theta^j)$. This power transformation arises naturally from the structure of the reciprocal gamma distribution. The condition $j < -\beta$ ensures that the gamma function remains finite.

When $\beta \in [-2, -1)$ the variance will be infinite. Therefore, this distribution can be used as the representative of the lack knowledge about the parameter when there are some belief about its mean value. In this case, only need the subjective mean value, then parameters $\beta, \chi, \psi$ can be obtained by derive (3.25) as follows

$$E(\Theta) = \mu = -\frac{\chi}{2(\beta+1)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3.27)$$

$$\chi = -2\mu(\beta+1), \beta \in [-2,-1), \psi = 0$$

**Inverse Gaussian distribution** with parameters $\mu$ and $\chi$ has density function

$$\pi(\theta) = \sqrt{\frac{\chi}{2\pi\theta}} e^{\left(-\frac{1}{2}\frac{\chi(\theta-\mu)^2}{\mu^2\theta}\right)}, \theta > 0, \mu > 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3.28)$$

The predictive distribution with $\beta = -\frac{1}{2}, \chi > 0, \psi > 0$, let $\psi = \frac{\chi}{\mu^2} (\mu > 0)$ and $\chi = \frac{\mu^3}{\sigma^2}$. It is

$$g_{-\frac{1}{2},\chi,\psi}(y) = \frac{\sqrt{\frac{\chi}{\psi}} e^{\sqrt{\chi\psi}\left[1-\left(1+\frac{2}{\psi}\log(1+y)\right)^{\frac{1}{2}}\right]}}{(1+y)\left(1+\frac{2}{\psi}\log(1+y)\right)^{\frac{1}{2}}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3.29)$$

**Reciprocal Inverse Gaussian distribution** with parameters $\psi$ and $\chi$ has density function

$$\pi(\theta) = \frac{1}{\sqrt{2\pi\psi\theta}} e^{-\frac{1}{2}\left(\frac{\chi}{\theta}+\psi\theta\right)}, \theta > 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3.30)$$

The predictive distribution with $\beta = \frac{1}{2}, \chi > 0, \psi > 0$, let $\psi = \frac{\chi}{\mu^2} (\mu > 0)$ and $\chi = \frac{\mu^3}{\sigma^2}$. It is

$$g_{\frac{1}{2},\chi,\psi}(y) = \sqrt{\frac{\chi}{\psi}} \frac{e^{\sqrt{\chi\psi}}}{(1+y)} e^{-\sqrt{\chi\psi}\left(1+\frac{2}{\psi}\log(1+y)\right)^{\frac{1}{2}}} \frac{\left[1+\chi\psi\left(1+\frac{2}{\psi}\log(1+y)\right)^{\frac{1}{2}}\right]}{\left(1+\frac{2}{\psi}\log(1+y)\right)} \ldots\ldots\ldots \quad (3.31)$$

# 4. RESULTS

## 4.1 Descriptive Analysis

The total fire insurance claim in Denmark is 7335.485 with the maximum value is 263.250 and the minimum value is 1.00. The dataset has a kurtosis value of 483.764 larger than the kurtosis value of the normal distribution. This high kurtosis indicates that the fire insurance claim data is leptokurtic, meaning the distribution has a sharper peak and heavier tails compared to the normal distribution. To better visualize the tail behavior of the data, a histogram was generated using RStudio software (see Figure 4.1).

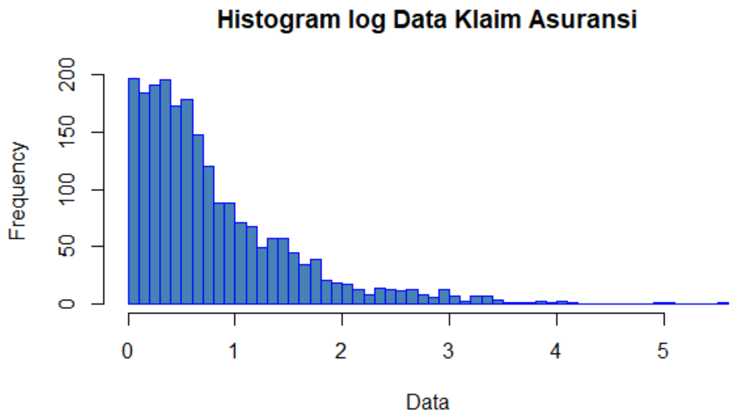


**Figure 4.1**. Danish fire insurance histogram

Histogram on Figure 4.1 shows that the data is not normally distributed with the right tail moving towards zero slowly, which indicates the data is heavy-tailed.

## 4.2 Modeling the Tail of Claim Distribution

Reinsurance claim is large type of claim, so modeling the tail can be done by using the extreme value theory approach. It is easier to use modeling of the excess value with a certain threshold value. These values asymptotically follow the Generalized Pareto Distribution (GPD). In this study, the focus is on the case of GPD having a positive shape parameter ($\xi > 0$) which corresponds to Pareto distributions. The threshold value is obtained using the mean excess plot [6] with RStudio software (see Figure 4.2).

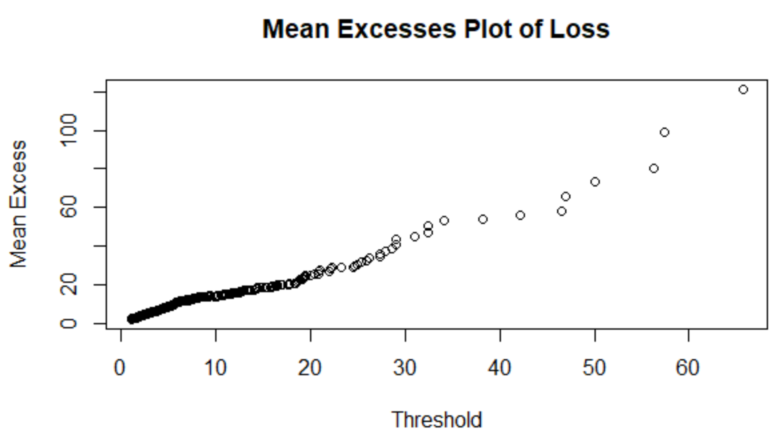


**Figure 4.2**. Mean excess plot of losses

Determining the threshold value with mean excess plot is usually difficult and more subjective. Often it is necessary to select several values and then check the goodness of fit. Here, the selection of the threshold is carried out using the help of the $0.999^{\text{th}}$ quantile estimation and the AIC value of each threshold. Two thresholds are obtained, they are 10 and 20.

The data distribution below the threshold value of 10 is GPD with the parameters are $\xi = 0.1874, \mu = 0.9829, \sigma = 1.0613$. The density function is

$$\boldsymbol{f_{\mu,\sigma,\xi}(x) = \frac{1}{1.0613}\left(1 + \frac{0.1874(x - 0.9829)}{1.0613}\right)^{\left(-\frac{1.1874}{0.1874}\right)}, 1 < x < 10}$$

And the probability of occurance of data that exceeds the threshold value of 10 is

$$\boldsymbol{p^u = P(x > 10) = 0.02216}$$

The data distribution below the threshold value of 20 is also GPD with the parameters are $\xi = 0.39933, \mu = 0.99194, \sigma = 1.0189$. The density function is

$$\boldsymbol{f_{\mu,\sigma,\xi}(x) = \frac{1}{1.0189}\left(1 + \frac{0.39933(x - 0.99194)}{1.0189}\right)^{\left(-\frac{1.39933}{0.39933}\right)},}$$
$$\boldsymbol{1 < x < 20}$$

And the probability of occurance of data that exceeds the threshold value of 10 is

$$\boldsymbol{p^u = P(x > 20) = 0.0126423}$$

From the probability values above, it can be said that for threshold value of 10 there is 2.216% possibility of the claim data exceeding the threshold value. And for the threshold 20 there is possibility of 1.264%.

### 4.3 Bayesian Analysis for Data above Threshold

The Bayesian estimation can model the lack of prior information in two ways. And in this paper will use the reciprocal gamma distribution with infinite variance and finite mean to model the vague prior belief about the parameter estimation [18]. Assume that the mean value and coefficient variance are $\mu = 1.5$ and 0.3 respectively jointly with $\beta = -2$, the calculation will be presented in Table 4.1.

**Table 4.1**. Parameter Values for Prior and Posterior Distribution Θ

| No | Prior Distribution | u | Parameters | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | | $\beta$ | $\chi$ | $\psi$ | $\beta'$ | $\chi'$ | $\psi'$ |
| 1 | Inverse Gaussian | 10 | $-\frac{1}{2}$ | 16.67 | 7.41 | 108.5 | 16.67 | $2\sum\log(y_i + 1) + 7.41$ |
| | | 20 | $-\frac{1}{2}$ | 16.67 | 7.41 | 35.5 | 16.67 | $2\sum\log(y_i + 1) + 7.41$ |
| 2 | Reciprocal Inverse Gauusian | 10 | $\frac{1}{2}$ | 16.67 | 7.41 | 109.5 | 16.67 | $2\sum\log(y_i + 1) + 7.41$ |
| | | 20 | $\frac{1}{2}$ | 16.67 | 7.41 | 36,5 | 16.67 | $2\sum\log(y_i + 1) + 7.41$ |

Based on Table 4.1, it can be calculated the expected value and variance of each prior distribution used as follows

**Table 4.2**. The Values of Expected and Variance of Posterior Distribution Θ

| No | Prior Distribution | $u = 10$ | | $u = 20$ | |
|---|---|---|---|---|---|
| | | $\hat{\theta}(y_1, \dots, y_k)$ (Expected) | Variance | $\hat{\theta}(y_1, \dots, y_k)$ (Expected) | Variance |
| | Inverse Gaussian | 1.597274 | 0.021436 | 1.716022 | 0.064854 |

| | | | | | |
|---|---|---|---|---|---|
| 2 | Reciprocal Inverse Gauusian | 1.610694 | 0.021632 | 1.753815 | 0.066592 |

Furthermore, Bayesian estimation of the posterior mean number of excesses is still needed to calculate reinsurance premiums. Based on the assumption that this paper used the modeling vague prior belief, the help of reciprocal gamma distribution with the mean equal to 2 (refers to [10]) and the infinite variance $\beta = -2$. From (3.27) the parameters of posterior distribution of the number of excesses obtained

**Table 4.3**. Parameters Estimation of Posterior Distribution $\Lambda^u$

| No | Prior Distribution | u | Parameters | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | | $\beta = \delta$ | $\chi = v$ | $\psi = \tau$ | $\delta'$ | $v'$ | $\tau'$ |
| | Reciprocal Gamma | 10 | -2 | 4 | 0 | 107 | 4 | 20 |
| | | 20 | -2 | 4 | 0 | 34 | 4 | 20 |

Based on Table 4.3, it can be calculated the expected value of the number of excesses as follows

**Table 4.4**. The Expected Values of Posterior Distribution $\Lambda^u$

| No | Prior Distribution | Threshold | $\hat{\lambda}^u(n_1, \dots, n_k)$ (Expected ) |
|---|---|---|---|
| 1 | Reciprocal Gamma | 10 | 10.7188 |
| | | 20 | 3.4595 |

Lastly the reinsurance premiums can be calculated by substituting $\hat{\theta}(y_1, \dots, y_k)$ and $\hat{\lambda}^u(n_1, \dots, n_k)$ into (3.17). The following table is the results of the estimated reinsurance premium prices from 2 different threshold.

**Table 4.5**. The Premium Estimation of Fire Insurance Claim Denmark Period 1980-1990

| No | Prior Distribution | Premiums | |
|---|---|---|---|
| | | $u = 10$ | $u = 20$ |
| 1 | Inverse Gaussian | 3.977272 | 1.15062 |
| 2 | Reciprocal Inverse Gaussian | 3.889871 | 1.60392 |

## 4.4 The Premium Illustration for a Reinsurance Contract

The premiums that have been obtained are compared with the expected benefits covered by the reinsurance company. The claim paid by the reinsurance company is the amount of the claims minus the threshold value. In this case, the insurance company can claim a claim to the reinsurance company if the amount of the claim exceeds the threshold value. The total reinsurance claims for the threshold value of 10 is 2624.914 (in million Krone Denmark) and the expected benefit that may be covered is 2.216% of the total reinsurance claims. This is the same for threshold value 20 is 1607.037 (in million Krone Denmark), the expected benefit that may be covered is 1.264% of the total reinsurance claims.

**Table 4.6** The Illustration of Reinsurance Policy Contract Terms

| Description | | Calculation | |
|---|---|---|---|
| | | $u = 10$ | $u = 20$ |
| Claim period (year) | | 10 | 10 |
| Single premium (million Krone Denmark) | Inverse Gaussian | 3.977272 | 1.15062 |
| | Reciprocal Inverse Gaussian | 3.889871 | 1.60392 |
| Total reinsured claims (million Krone Denmark) | | 2624.914 | 1607.037 |
| Probability Tail | | 2.167% | 1.264% |
| Expected Benefit (million Krone Denmark) | | 58.168 | 20.313 |

Premium values for thresholds 10 and 20 are smaller than the expected benefit which will be obtained by the insurance company. It shows that the reinsurance premiums are logical and not burdensome for the insurance company. And for the reinsurance company, a lower threshold makes the probability of a claim occur bigger.

## 5. CONCLUSION

Based on this research, extreme value theory is one of the formal approaches that can be used in modeling heavy-tailed distributed data. In the calculation of reinsurance premiums, the EVT modeling used is modeling the excess value with a certain threshold. Using the mean excess plot, the threshold values of 10 and 20 are obtained. To model the data above the threshold or in this study called data excess, the prior Inverse Gaussian distribution and the reciprocal Inverse Gaussian distribution and apply them to the calculation of reinsurance premiums. In the premium results obtained, the two thresholds can be used as options for reinsurance companies as insurers if they want to make a reinsurance policy contract. Likewise for insurance companies, the higher the threshold value, the cheaper the premium price and the smaller the probability for claims to occur.

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