Mathematics Model SAHTR for the Number of Drug Abusers with Economic and Educational Factors

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Abstract

This study develops a mathematical model of drug abuse involving susceptible (S), light users (A), heavy users (H), treatment (T), and recovered (R) compartments, incorporating economic and educational factors. The analysis includes determining equilibrium points, assessing their stability, calculating the basic reproduction number, and performing numerical simulations using the Runge–Kutta Fehlberg method. Results show that the model yields two equilibrium points: drug-free and endemic. Both are stable when the inflow rate into the light-user compartment—affected by effective contact rate, anti-drug campaign effectiveness, and economic conditions—exceeds the outflow rate. Numerical simulations confirm the analytical findings and illustrate that reducing interactions between vulnerable individuals and drug users, strengthening anti-drug campaigns, and improving economic conditions can diminish the potential spread of drug abuse.

Keywords: Mathematical Model, Drug Abuse, Runge-Kutta Fehlberg, Economic Factor, Educational Factor

1. INTRODUCTION

According to the Republic of Indonesia Law Number 35 of 2009 concerning Narcotics, narcotics are substances or drugs derived from plants or non-plants which cause decreased consciousness, reduce or eliminate pain, and cause dependence [24]. Narcotics, commonly called drugs, are substances that are used legally and are used as medicine in the medical world, but are now often misused by many people [23]. Regular drug abuse beyond medical necessity can lead to physical, psychological, and social dysfunction and can lead to addiction and dependency [19]. Currently, drug abuse in Indonesia continues to increase, not only among the upper economic status but also among the lower and middle economic status [16]. Even teenagers and children are involved in drug abuse. The World Drug Report 2023 reports that the number of drug abusers has increased by 12 million people from the previous year to 296 million, equivalent to 5,8% of the global population aged 15–64 years [25]. Meanwhile, the 2023 National Survey on Drug Abuse reports that the prevalence of drug abuse is 1,73%, equivalent to 3,3 million Indonesians aged 15–64 years [11].

This increase is inseparable from various factors influencing drug abuse behavior in society, one of which is economic factors. Based on data on the number of drug abusers by occupation in 2023 [10], drug abusers were predominantly unemployed, with 784 individuals. This was followed by the

self-employed (296 individuals), private sector workers (265 individuals), and laborers (115 individuals). These four groups reflect a relatively vulnerable economic situation, ranging from unemployment, high work pressure, irregular income, and low-wage employment. In contrast, groups with more stable conditions, such as the Indonesian National Armed Forces (59 individuals), civil servants (28 individuals), and even doctors and doctoral graduates had no abusers. This fact reinforces that economic factors, such as poverty and homelessness, fatigue and long working hours, and child labor, influence individuals to use drugs [5]. Besides economic factors, education is equally important. In this study, education represents external efforts to prevent drug abuse. This education is realized in the form of an anti-drug campaign and strengthening self-psychology through counseling. The anti-drug campaign related to the dangers of drugs has significant benefits in terms of changing the knowledge of campaign participants, which is indicated by a decrease in the percentage of the less than knowledge category from 33,3% to 6,06%, an increase in the percentage of the sufficient knowledge category from 57,57% to 73,73%, and good knowledge from 9,09% to 20,20% (Kumalasari, Rahmah, & Hastuti, 2022). Then, the effectiveness of strengthening self-psychology through counseling is able to change the quality of life of former drug addicts, which has an impact on physical health, psychological conditions, life satisfaction, social connectedness, and connectedness with the surrounding environmental conditions, so that former drug addicts can engage in productive activities and forget about drugs [14].

Drug abuse can spread through social interactions, where individuals involved in drug abuse can encourage others to use drugs, allowing for mathematical modeling of this problem. Mathematical models make it easier to solve real-life problems [3]. Various mathematical models have been developed by previous researchers: the SCLHTR mathematical model with the influence of family education and public health education [18], the SURS mathematical model by adding educational factors so that the population is divided into groups that receive education and those that do not receive education [22], the SAHTR mathematical model by providing optimal control in the form of the effects of implementing anti-drug campaigns and strengthening self-psychology through counseling [3], the mathematical model of drug abuse spreading, incorporating a death rate by addiction [4], the mathematical model of drug abuse reduction strategies includes treatment type and risks level [1], the mathematical model for the importance of early awareness strategy on the dynamics of drug addiction [2], the mathematical model for the dynamics of crime and substance abuse within a population that integrates social interactions, rehabilitation efforts, and relapse probabilities [20]. Different from previous studies, the novelty of this study is considering economic factors and educational factors simultaneously to develop a mathematical model of the number of drug abusers and using the SAHTR model.

2. MATHEMATICAL MODEL CONSTRUCTION

The SAHTR model divides the population into five compartments: susceptible (S), light users (A), heavy users (H), treatment (T), and recovered (R). The assumptions used in developing this mathematical model begin with new individuals entering the susceptible compartment. Individuals in the susceptible compartment can move to the light users compartment due to social interactions with light and heavy users, influenced by economic factors and the low effectiveness of anti-drug campaigns. Furthermore, individuals in the light users compartment can experience an increase in dosage or frequency of use, thus moving to the heavy users compartment. Then, abusers can undergo rehabilitation or enter the treatment compartment. In the rehabilitation process, there are two possibilities: individuals can recover and move to the recovered compartment or fail to undergo rehabilitation (dropout) and return to the light users compartment. Individuals in the recovered class do not fully recover; they can return to the susceptible compartment or experience a relapse and enter the light users compartment. Each compartment experiences natural death, but individuals in

the heavy users compartment death due to drug dependence. Based on these assumptions, a flow diagram of the SAHTR model is illustrated in Figure 2.1.
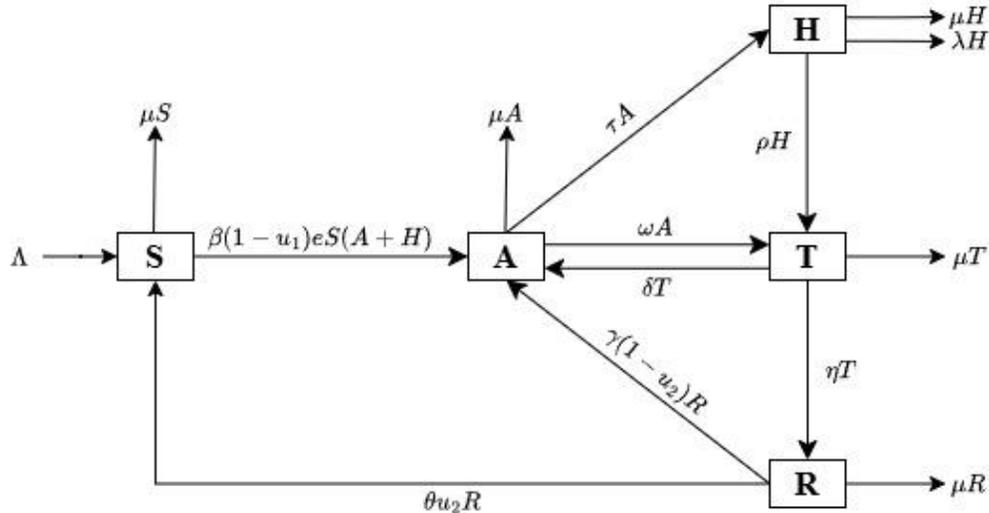

Figure 2.1. Flow diagram of drug abusers with economic and educational factors

Through the flow diagram depicted in Figure 2.1, a system of nonlinear differential equations is obtained and presented below.

$$\frac{dS}{st} = \Lambda - \beta(1 - u_1)eS(A + H) - \mu S + \theta u_2 R$$
$$\frac{dA}{dt} = \beta(1 - u_1)eS(A + H) + \gamma(1 - u_2)R + \delta T - (\mu + \tau + \omega)A$$
$$\frac{dH}{dt} = \tau A - (\mu + \lambda + \rho)H \qquad (2.1)$$
$$\frac{dT}{dt} = \omega A + \rho H - (\mu + \eta + \delta)T$$
$$\frac{dR}{dt} = \eta T - (\gamma(1 - u_2) + \mu + \theta u_2)R$$

where $S(t)$ is the number of individuals vulnerable to using drugs at time t, $A(t)$ is the number of light drug abusers at time t, $H(t)$ is the number of heavy drug abusers at time t, $T(t)$ is the number of drug abusers undergoing rehabilitation at time t, and $R(t)$ is the number of drug abusers recovering from drug dependence at time t. For every time t, the number of individuals in each compartment is non-negative, that is $S(t) \geq 0, A(t) \geq 0, H(t) \geq 0, \; T(t) \geq 0$, and $R(t) \geq 0$.

The parameters in Figure 2.1 are explained as follows and all parameters are positive:

Λ	:	the recruitment rate of individuals aged 15–64 years.
β	:	the effective contact rate between vulnerable individuals and abusers.
μ	:	the natural death rate, λ is the death rate due to drug dependence.
ω	:	the rate of movement of individuals from the light users compartment (A) to the treatment compartment (T).
ρ	:	the rate of movement of individuals from the heavy users (H) to the treatment compartment (T).
δ	:	the rate of movement of individuals from the treatment compartment (T) to the light users compartment (R).
η	:	the rate of movement of individuals from the treatment compartment (T) to the recovered compartment (R).

γ : the rate of movement of individuals from the recovered compartment (R) to the light users compartment (A).

τ : the rate of movement of individuals from the light users compartment (A) to the heavy users (H).

θ : the rate of movement of individuals from the recovered compartment (R) to the susceptible (S).

u_1 : the effectiveness of the anti-drug campaign ($0 < u_1 < 1$). As u_1 approaches 1, indicating a more effective campaign, the number of individuals influenced to use drugs decrease.

u_2 : the effectiveness of strengthening self-psychology through counseling ($0 < u_2 < 1$). As the effectiveness of this counseling increases, a greater number of individuals return to the susceptible compartment (T).

e : an economic factor that influences drug abuse.

3. RESULT AND DISCUSSION

3.1. Equilibrium Point

The equilibrium point in the SAHTR model is obtained when all equations in system (2.1) are zero, namely $\frac{dS}{dt} = 0, \frac{dA}{dt} = 0, \frac{dH}{dt} = 0, \frac{dT}{dt} = 0$, and $\frac{dR}{dt} = 0$. To simplify the process of determining the equilibrium point, the equations in system (2.1) are reduced by defining $m_1 = \mu + \tau + \omega, m_2 = \mu + \lambda + \rho, m_3 = \mu + \eta + \omega$, and $m_4 = \gamma(1 - u_2) + \mu + \theta u_2$, so that the following is obtained.

$$\Lambda - \beta(1 - u_1)eS(A + H) - \mu S + \theta u_2 R = 0$$
$$\beta(1 - u_1)eS(A + H) + \gamma(1 - u_2)R + \delta T - m_1 A = 0$$
$$\tau A - m_2 H = 0 \tag{3.1}$$
$$\omega A + \rho H - m_3 T = 0$$
$$\eta T - m_4 R = 0$$

The system equation (3.1) produces two equilibrium point solutions, namely the drug-free equilibrium point and the endemic equilibrium point. The drug-free equilibrium point is a condition where no individuals abuse drugs, or in other words, there are no light abusers or heavy abusers ($A = 0$ and $H = 0$). Consequently, no one is undergoing rehabilitation ($T = 0$) and recovering from drug dependence ($R = 0$). Thus, a drug-free equilibrium point is obtained, as shown in equation (3.2).

$$T_0(S, A, H, T, R) = \left(\frac{\Lambda}{\mu}, 0, 0, 0, 0\right) \tag{3.2}$$

Then, the endemic equilibrium point is a condition where there are still drug abusers in the population, whether at a mild or severe level. The endemic point is shown in equation (3.3).

$$T_1(S, A, H, T, R) = (S^*, A^*, H^*, T^*, R^*) \tag{3.3}$$

where

$$S^* = \frac{-(\gamma\eta\rho\tau(1-u_2)+\gamma\eta m_2\omega(1-u_2)+\delta m_4\rho\tau+\delta m_2 m_4\omega-m_1 m_2 m_3 m_4)}{\beta e(1-u_1)(m_2 m_3 m_4 + m_3 m_4\tau)}$$

$$A^* = \frac{m_2 m_3 m_4 R^*}{\eta\rho\tau + m_2\eta\omega}$$

$$H^* = \frac{m_3 m_4 \tau R^*}{\eta\rho\tau + m_2\eta\omega}$$

$$T^* = \frac{m_4 R^*}{\eta}$$

$$R^* = \frac{\Lambda\eta\rho\tau + \Lambda m_2\eta\omega - S^*\mu(\eta\rho\tau + m_2\eta\omega)}{S^*\beta(1-u_1)e(m_2 m_3 m_4 + m_3 m_4\tau) - \theta u_2\eta\rho\tau - \theta u_2 m_2\eta\omega}$$

3.2. Equilibrium Point Stability Analysis

The stability of the obtained equilibrium point will be analyzed using the eigenvalues of the Jacobian matrix of system (2.1). The following is an analysis of the stability of the drug-free equilibrium point of system (2.1). First, the Jacobian matrix of system (2.1) is formed to obtain

$$J(T_0) = \begin{bmatrix} -\mu & -\frac{\beta(1-u_1)e\Lambda}{\mu} & -\frac{\beta(1-u_1)e\Lambda}{\mu} & 0 & \theta u_2 \\ 0 & \frac{\beta(1-u_1)e\Lambda}{\mu} - (\mu+\tau+\omega) & \frac{\beta(1-u_1)e\Lambda}{\mu} & \delta & \gamma(1-u_2) \\ 0 & \tau & -(\mu+\lambda+\rho) & 0 & 0 \\ 0 & \omega & \rho & -(\mu+\eta+\delta) & 0 \\ 0 & 0 & 0 & \eta & -(\gamma(1-u_2)+\mu+\theta u_2) \end{bmatrix}$$

Several eigenvalues of the matrix $J(T_0)$ are obtained, namely:

$\psi_1 = -\mu$

$\psi_2 = \frac{\Lambda\beta e(1-u_1)}{\mu} - (\mu+\tau+\omega)$

$\psi_3 = -(\mu+\lambda+\rho)$

$\psi_4 = -(\mu+\eta+\delta)$

$\psi_5 = -(\gamma(1-u_2)+\mu+\theta u_2)$

The drug-free equilibrium point $T_0(S, A, H, T, R) = (\frac{\Lambda}{\mu}, 0, 0, 0, 0)$ will be stable when $\psi_1 < 0, \psi_2 < 0, \psi_3 < 0, \psi_4 < 0$, and $\psi_5 < 0$ are satisfied, thus obtaining

1. $-\mu < 0 \Rightarrow \mu > 0$
2. $\frac{\Lambda\beta e(1-u_1)}{\mu} - (\mu+\tau+\omega) < 0 \Rightarrow \Lambda\beta e(1-u_1) < \mu(\mu+\tau+\omega)$
3. $-(\mu+\lambda+\rho) < 0 \Rightarrow \mu+\lambda+\rho > 0$
4. $-(\mu+\eta+\delta) < 0 \Rightarrow \mu+\eta+\delta > 0$
5. $-(\gamma(1-u_2)+\mu+\theta u_2) < 0 \Rightarrow \gamma(1-u_2)+\mu+\theta u_2 > 0$

The value of ψ_2 is negative if $\Lambda\beta e(1-u_1) < \mu(\mu+\tau+\omega)$. This indicates that there will be no endemic if combination of the recruitment rate of individuals (Λ) and the effective contact rate between vulnerable individuals and abusers (β) and an economic factor that influences drug abuse (e) and success of campaign influence ($u_1 \to 1$) are less than the natural death rate (μ) and the rate of movement of individuals from the light to heavy users compartment (τ) and the rate of movement of individuals from the light users to treatment compartment (ω).

Next, analyze the stability of the endemic equilibrium point of system (2.1).

$$J(T_1) = \begin{bmatrix} -\beta(1-u_1)e(A^*+H^*)-\mu & -\beta(1-u_1)eS^* & -\beta(1-u_1)eS^* & 0 & \theta u_2 \\ \beta(1-u_1)e(A^*+H^*) & \beta(1-u_1)eS^* - (\mu+\tau+\omega) & \beta(1-u_1)eS^* & \delta & \gamma(1-u_2) \\ 0 & \tau & -(\mu+\lambda+\rho) & 0 & 0 \\ 0 & \omega & \rho & -(\mu+\eta+\delta) & 0 \\ 0 & 0 & 0 & \eta & -(\gamma(1-u_2)+\mu+\theta u_2) \end{bmatrix}$$

Several eigenvalues of the matrix $J(T_1)$ are obtained, namely:

$\psi_1 = -\beta(1-u_1)e(A^*+H^*)-\mu$

$\psi_2 = \beta(1-u_1)eS^* - (\mu+\tau+\omega)$

$\psi_3 = -(\mu+\lambda+\rho)$

$\psi_4 = -(\mu + \eta + \delta)$
$\psi_5 = -(\gamma(1 - u_2) + \mu + \theta u_2)$

The endemic equilibrium point $T_1(S, A, H, T, R) = (S^*, A^*, H^*, T^*, R^*)$ will be stable when $\psi_1 < 0, \psi_2 < 0, \psi_3 < 0, \psi_4 < 0$, and $\psi_5 < 0$ are satisfied, thus obtaining

1. $\beta(1 - u_1)eS^* - (\mu + \tau + \omega) < 0$
2. $-(\mu + \lambda + \rho) < 0$
3. $-(\mu + \eta + \delta) < 0$
4. $-(\gamma(1 - u_2) + \mu + \theta u_2) < 0$

The value of ψ_2 is negative if

$\beta e(1 - u_1)S^* = \frac{-(\gamma\eta\rho\tau(1-u_2)+\gamma\eta m_2\omega(1-u_2)+\delta m_4\rho\tau+\delta m_2 m_4\omega-m_1 m_2 m_3 m_4)}{(m_2 m_3 m_4+m_3 m_4\tau)} < (\mu + \tau + \omega)$, or $-(\gamma\eta\rho\tau(1 - u_2) + \gamma\eta m_2\omega(1 - u_2) + \delta m_4\rho\tau + \delta m_2 m_4\omega - m_1 m_2 m_3 m_4) < (\mu + \tau + \omega)(m_2 m_3 m_4 + m_3 m_4\tau)$. This indicates that endemic will occurs if $(u_2 \to 0)$ and the rate of movement of individuals to the light user from the susceptible, treatment, and recoverd compartments are greater than the natural death rate (μ).

3.3. Basic Reproduction Number

The basic reproduction number (R_0) is the average number of secondary cases that arise as a result of one primary case in a disease-susceptible population, which is useful for measuring the potential for disease spread in a population (van den Driessche & Watmough, 2008). The basic reproduction number can be calculated using a next generation matrix [13]. In system (2.1), infections are in the light users (A) and heavy users (H) compartments. Thus, the differential equation under consideration is

$\frac{dA}{dt} = \beta(1 - u_1)eS(A + H) + \gamma(1 - u_2)R + \delta T - (\mu + \tau + \omega)A$

$\frac{dH}{dt} = \tau A - (\mu + \lambda + \rho)H$

Then, the Jacobian matrix of $\mathbb{F}$ and $\mathbb{V}$ are constructed, where F_i is the rate of emergence of new cases that increase the population of disease compartment i, and V_i is the rate of population decline in the disease compartment due to disease progression, death, or recovery. Thus, we obtain:

$F_1 = \beta(1 - u_1)eS(A + H) + \gamma(1 - u_2)R + \delta T$ and $F_2 = \tau A$

$V_1 = (\mu + \tau + \omega)A$ and $V_2 = (\mu + \lambda + \rho)H$

The Jacobian matrix of F is

$\mathbb{F} = \begin{bmatrix} \frac{\partial F_1(T_0)}{\partial A} & \frac{\partial F_1(T_0)}{\partial H} \\ \frac{\partial F_2(T_0)}{\partial A} & \frac{\partial F_2(T_0)}{\partial H} \end{bmatrix} = \begin{bmatrix} \beta(1 - u_1)eS_0 & \beta(1 - u_1)eS_0 \\ \tau & 0 \end{bmatrix}$, where $S_0 = \frac{\Lambda}{\mu}$

The Jacobian matrix of V is

$\mathbb{V} = \begin{bmatrix} \frac{\partial V_1(T_0)}{\partial A} & \frac{\partial V_1(T_0)}{\partial H} \\ \frac{\partial V_2(T_0)}{\partial A} & \frac{\partial V_2(T_0)}{\partial H} \end{bmatrix} = \begin{bmatrix} \mu + \tau + \omega & 0 \\ 0 & \mu + \lambda + \rho \end{bmatrix}$ and $\mathbb{V}^{-1} = \begin{bmatrix} \frac{1}{\mu+\tau+\omega} & 0 \\ 0 & \frac{1}{\mu+\lambda+\rho} \end{bmatrix}$

Therefore,

$$\mathbb{F}\mathbb{V}^{-1} = \begin{bmatrix} \frac{\beta eS_0(1 - u_1)}{\mu + \tau + \omega} & \frac{\beta eS_0(1 - u_1)}{\mu + \lambda + \rho} \\ \frac{\tau}{\mu + \tau + \omega} & 0 \end{bmatrix}$$

The basic reproduction number (R_0) is obtained from the spectral radius or the largest eigenvalue of the matrix $\mathbb{FV}^{-1}$, thus obtaining:

$$R_0 = \frac{\dfrac{\beta e S_0(1-u_1)}{\mu+\tau+\omega} + \sqrt{\dfrac{\left(\beta e S_0(1-u_1)\right)^2}{(\mu+\tau+\omega)^2} + \dfrac{4\tau\left(\beta e S_0(1-u_1)\right)}{(\mu+\tau+\omega)(\mu+\lambda+\rho)}}}{2}$$

If $R_0 < 1$, then drug abuse will gradually disappear because each abuser only causes less than one new case. Otherwise, if $R_0 > 1$, then drug abuse will continue to spread because one drug abuser can spread their influence to more than one other individual.

3.4. Numerical Simulation

Based on real problems in Indonesia, numerical simulation for system equation (2.1) is carried out using initial values obtained from [6], [8], and [12], namely $S(0) = 180369250, A(0) = 4855123, H(0) = 3683848, T(0) = 4232$, and $R(0) = 2847$. And using parameter values obtained from various literature sources [3], [7], [18], [21] and several assumptions, namely $\Lambda = 1000, \beta = 0.0009, \mu = 0.02, \tau = 0.03, \lambda = 0.3, \omega = 0.3, \rho = 0.421, \delta = 0.78, \eta = 0.25, \gamma = 0.3, \theta = 0.7, u_1 = 0.5, u_2 = 0.5$, and $e = 0.1$. Numerical simulations in this study were conducted using the Runge-Kutta Fehlberg (RKF45) method.

The initial values and parameters will be used to analyze the stability conditions for the drug-free equilibrium point and the endemic equilibrium point. This is done to determine whether the system will move towards the drug-free equilibrium point or the endemic equilibrium point. The drug-free equilibrium point will be stable if the following five conditions.

1. $\mu > 0$
   $\Rightarrow 0.02 > 0$ (fulfilled)
2. $\Lambda\beta e(1-u_1) < \mu(\mu+\tau+\omega)$
   $\Rightarrow 0.045 > 0.007$ (not fulfilled)
3. $\mu+\lambda+\rho > 0$
   $\Rightarrow 0.741 > 0$ (fulfilled)
4. $\mu+\eta+\delta > 0$
   $\Rightarrow 1.05 > 0$ (fulfilled)
5. $\gamma(1-u_2)+\mu+\theta u_2 > 0$
   $\Rightarrow 0.520 > 0$ (fulfilled)

Based on the calculations in points 1 through 5, one condition was not fulfilled, making the drug-free equilibrium unstable. Therefore, numerical calculations were performed to determine the stability conditions for the endemic equilibrium. The endemic equilibrium will be stable if the following five conditions are met.

1. $-\beta(1-u_1)e(A^*+H^*)-\mu < 0$
   $\Rightarrow -1.723 < 0$ (fulfilled)
2. $\beta(1-u_1)eS^*-(\mu+\tau+\omega) < 0$
   $\Rightarrow -0.3411 < 0$ (fulfilled)
3. $-(\mu+\lambda+\rho) < 0$
   $\Rightarrow -0.741 < 0$ (fulfilled)
4. $-(\mu+\eta+\delta) < 0$
   $\Rightarrow -1.050 < 0$ (fulfilled)
5. $-(\gamma(1-u_2)+\mu+\theta u_2) < 0$

$$\Rightarrow -0.520 < 0 \qquad \text{(fulfilled)}$$

Based on the calculations in points 1 to 5, all stability requirements are fulfilled, so the system is stable at the endemic equilibrium point, and the system will move towards an endemic state. Next, the endemic equilibrium point is calculated analytically using the previously mentioned parameter values. Based on the calculation results, the endemic equilibrium point is obtained as

$$T_1(S, A, H, T, R) = (1980, 22922.5, 928, 6921.4, 3327.6)$$

Then, the basic reproduction number is also calculated to support the stability of the endemic equilibrium point as an indicator of population distribution in the population.

$$R_0 = \frac{\dfrac{\beta e S_0(1 - u_1)}{\mu + \tau + \omega} + \sqrt{\dfrac{\left(\beta e S_0(1 - u_1)\right)^2}{(\mu + \tau + \omega)^2} + \dfrac{4\tau\left(\beta e S_0(1 - u_1)\right)}{(\mu + \tau + \omega)(\mu + \lambda + \rho)}}}{2} \approx 6.47 > 1$$

This value indicates that the distribution will persist because one drug abuser can spread their influence to more than one individual. This causes the number of drug abusers to increase over time, so that in the long term, drug abusers will not disappear. This result aligns with the analysis of the stability of the equilibrium point, which indicates that the endemic equilibrium point is stable and the system will move towards an endemic state.

To support the analytical results, numerical simulations were performed using MATLAB. The simulation results are shown in Figure 3.1 and Table 3.1.
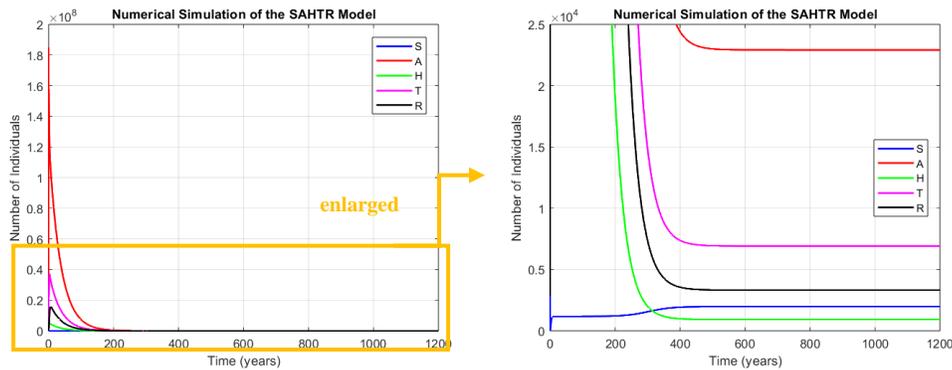

Figure 3.1. Numerical simulation for the SAHTR model

Table 3.1. The value of each compartment at the end of the period

Time	S	A	H	T	R
⋮	⋮	⋮	⋮	⋮	⋮
999.4	1980.0	22922.5	928.0	6921.4	3327.6
999.6	1980.0	22922.5	928.0	6921.4	3327.6
999.8	1980.0	22922.5	928.0	6921.4	3327.6
1000.0	1980.0	22922.5	928.0	6921.4	3327.6

Based on the numerical simulation results in Figure 3.1, it was found that the number of drug abusers remains constant at a certain value over the long term. This condition indicates that the system is approaching an endemic equilibrium point. The values for each compartment in Table 3.1 also indicate that the results align with the analytical results.

Based on these results, further simulations were conducted by varying the values of parameters deemed significant to examine their influence on the dynamics of the number of drug abusers. The

parameters varied in value include the effective contact rate (β), the effectiveness of the anti-drug campaign (u_1), and economic factors (e).

   To observe the effect of the effective contact rate (β) on changes in the number of drug abusers in the light users (A) and heavy users (H) compartments, numerical simulations were conducted using the aforementioned parameter values, but with the β value gradually decreased. The β values used were 0.09, 0.02, 0.0005, and 0.00001. The simulation results for changes in the β are shown in Figure 3.2.

(a) Graph of light users compartment (A) versus time t

(b) Graph of heavy users compartment (H) versus time t
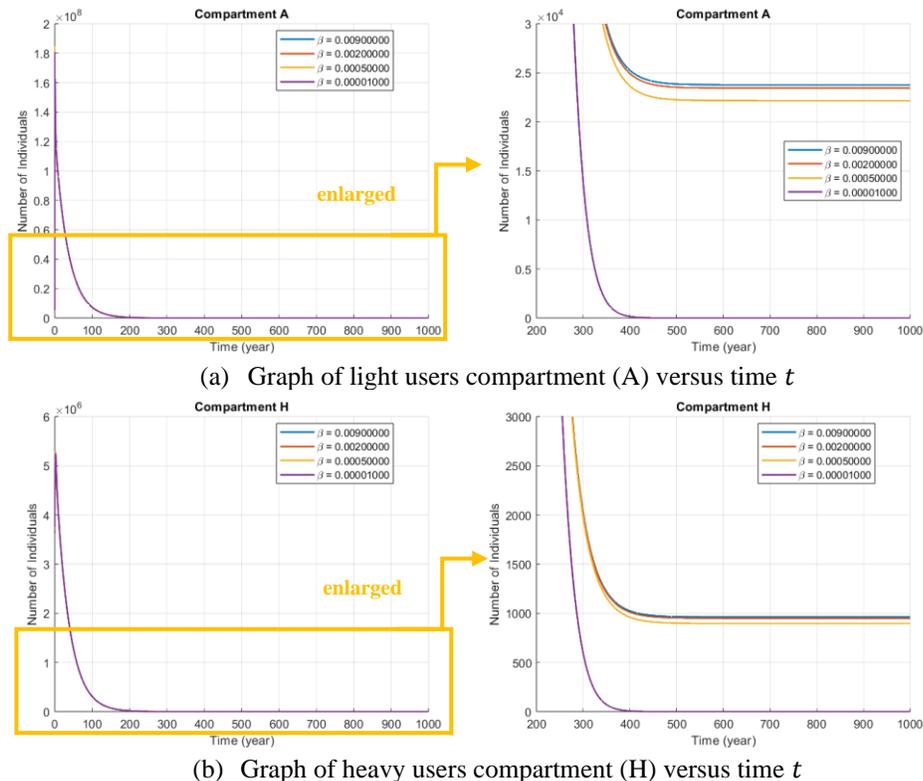
Figure 3.2. Dynamics of light users (A) and heavy users (H) compartment versus time t with $\beta = 0.009;\ 0.002; 0.0005; 0.00001$

   Based on the simulation results in Figure 3.2 (a), if the β value decreases, the probability of individuals transitioning to light users also decreases, resulting in a reduction in the number of individuals in the light users (A) compartment, which will eventually disappear in the long term. Consequently, the number of individuals with the potential to transition to heavy users will also decrease. Thus, the number of individuals in the heavy users (H) compartment will also decrease when the β value is reduced and will eventually disappear in the long term, as shown in Figure 3.2 (b). This finding aligns with previous research in [15], which indicates that negative peer environments and the influence of surrounding groups serve as entry points for individuals to try, learn about, and distribute drugs. This is also supported by the 2023 National Drug Abuse Survey report, where the majority of respondents stated that the source of drug acquisition came from friends, accounting for 84,5%, and the primary reason for first trying drugs was due to peer pressure/suggestion, with an average percentage exceeding 60% [11]. Therefore, a decrease in the β value can slow the rate of drug abuse spread. As a result, preventive efforts are necessary, such as enhancing the role of families and schools in character development and monitoring social

interactions to prevent vulnerable individuals from being influenced by negative environmental factors.

Next, observing the effect of the anti-drug campaign on changes in the number of drug abusers in the light users (A) and heavy users (H) compartments, a numerical simulation was performed using the parameters mentioned above, but the u_1 value was gradually increased. The values of u_1 used were 0.25, 0.5, 0.75, and 0.99. The simulation results for the changes in values are shown in Figure 3.3.
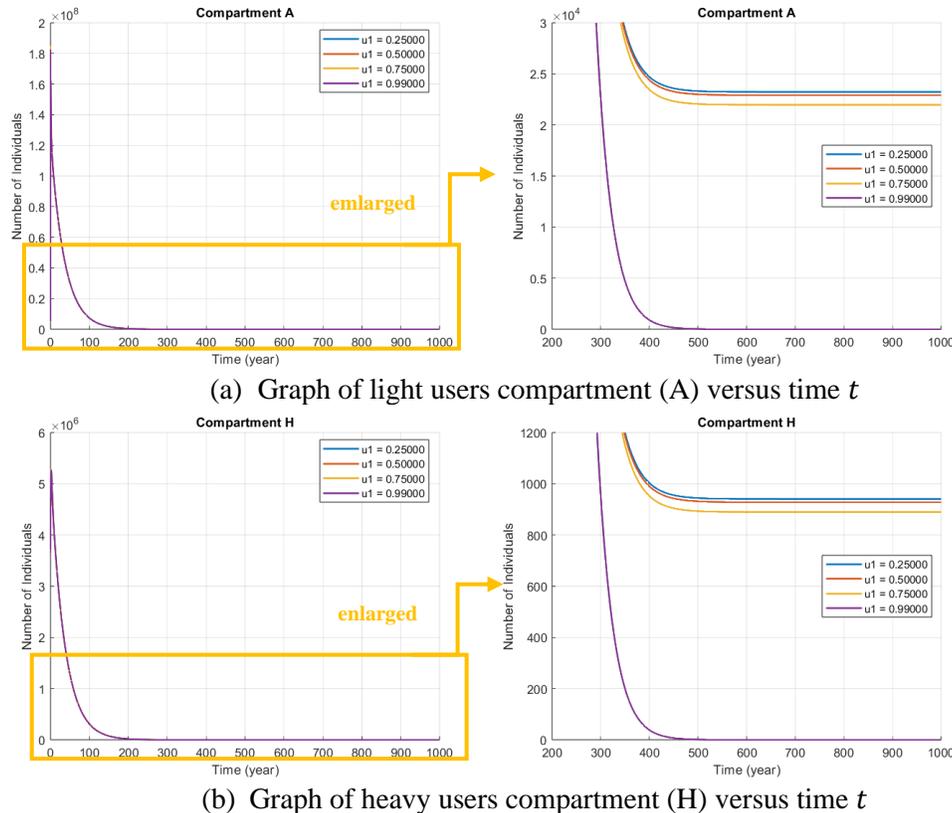

(a) Graph of light users compartment (A) versus time t

(b) Graph of heavy users compartment (H) versus time t

Figure 3.3. Dynamics of light users (A) and heavy users (H) compartment versus time t with $u_1 = 0.25;\ 0.5; 0.75; 0.99$

Based on the simulation results in Figure 3.3 (a), increasing the effectiveness of anti-drug campaigns can directly reduce the number of individuals vulnerable to becoming light users untul they eventually disappear. Meanwhile, heavy users will exist if individuals in the light users (A) compartment continue to use drugs repeatedly until they become dependent and move to the heavy users (H) compartment. Therefore, if the number of individuals in the light users (A) compartment is small, the number of individuals entering the heavy users (H) compartment will also be small. The simulation results in Figure 3.3 (b) show that when the value of u_1 is increased, the number of individuals in the heavy users (H) compartment also decrease or can be reduced. Thus, the more effective the anti-drug campaign as an initial prevention effort, the greater the likelihood of preventing drug abuse from becoming more severe. However, the 2023 National Drug Abuse Survey report indicates that among all respondents who are drug abusers, 63,5% have a high level of knowledge about the effect of drugs, 25% have a moderate level of knowledge, and only 11,5% have a low level of knowledge [11]. This data shows that a high level of knowledge about the effects of

drugs does not necessarily prevent someone from using drugs. There may be other contributing factors, such as peer pressure, social pressure, or economic factors. Thus, although numerical simulations of mathematical models show that increasing the effectiveness of anti-drug campaigns needs to be designed not only to convey information but also raise awareness and change the mindset of the individuals targeted for education.

Lastly, to observe the influence of economic factors (e) on changes in the number of drug abusers in the light users (A) and heavy users (H) compartments, a numerical simulation was conducted using the parameter values mentioned above, but with the value of e gradually reduced. The values of e used were 0.3, 0.1, 0.0009, and 0.00009. The simulation results for changes in the value of e are shown in Figure 3.4.
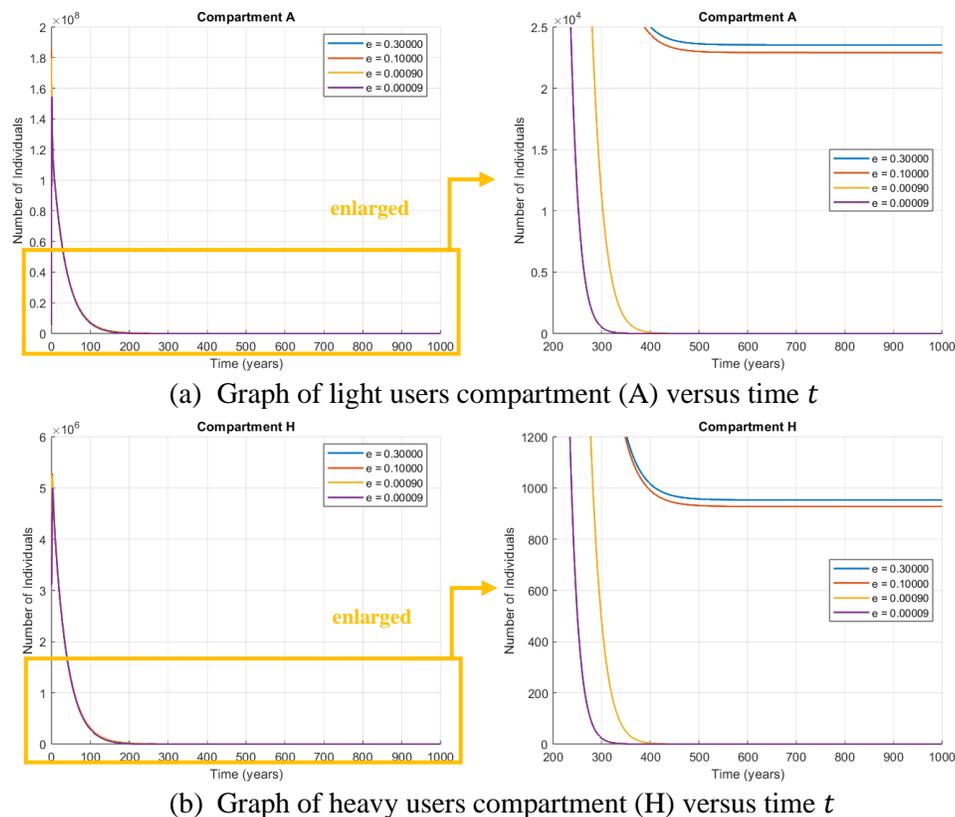

(a) Graph of light users compartment (A) versus time t

(b) Graph of heavy users compartment (H) versus time t

Figure 3.4. Dynamics of light users (A) and heavy users (H) compartment versus time t with e $=$ 0.3; 0.1; 0.0009; 0.00009

Based on the simulation results in Figure 3.4 (a), a decrease in economic factors can reduce the number of individuals who are light users. If the value of e decrease, the likelihood of vulnerable individuals becoming light users decreases, resulting in a decrease in the number of individuals in the light users compartment (A). As a result, the movement of individuals into the heavy users compartment (H) will also be smaller. The simulation results in Figure 3.4 (b) show that when the value of e is reduced, the number of individuals in the heavy users compartment (H) decreases. This means that if economic pressure decrease, the chance of individuals engaging in ab[use also decreases because they are no longer in a socially or psychologically vulnerable condition. In the 2021 National Survey on Drug Abuse, it was stated that 45,2% of drug users are classified as poor

[9]. This means that even though drugs are considered expensive, those living with economic constraints still use them, possibly due to life pressures, unemployment, or as a form of escape from economic problems. Therefore, improving a person's economic condition can reduce a person's desire to try drug use. Thus, efforts to improve community welfare, such as providing employment opportunities, need to be strengthened to reduce economic pressure as a risk factor to drug abuse.

4. CONCLUSION

This study constructs a mathematical model of the dynamics of the number of drug abusers with five compartments: susceptible (S), light users (A), heavy users (H), treatment (T), and recovered (R) to understand how economic and educational factors influence the dynamics of the number of drug abusers. From the model, two equilibrium points are obtained, namely the drug-free equilibrium point and the endemic equilibrium point. The drug-free equilibrium point and the endemic equilibrium point will be stable when the rate of entry of individuals into the light users compartment (A), which is influenced by the effective contact rate, the effectiveness of the anti-drug campaign, and economic factors is greater than the rate of exit of individuals from the light users (A) which is influenced by death, movement of individuals to the heavy users compartment (H), and entry of individuals to the treatment compartment (T). Then, from the model simulation, several facts were produced: the high effectiveness of the anti-drug campaign (u_1) and the decrease in the value of the economic factor (e), as well as the reduction in effective contact between vulnerable individuals and drug abusers (β) can reduce the potential for further spread and the number of drug abusers will decrease and eventually disappear in the population or it can be said that the system has reached a drug-dree condition.

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