Small Area Estimation for Gross Enrollment Rate at the College Level Using a Hierarchical Bayes Approach

Muh. Taslim Hakim[1*], Azka Ubaidillah[2]
[12]Politeknik Statistika STIS
Email*: 112112196@stis.ac.id[1], azka@stis.ac.id[2]*
*Corresponding author

Abstract

Quality education is one of the goals of the Sustainable Development Goals (SDGs) aimed at improving human resources. According to the March 2023 Susenas, participation at the college level has the lowest Gross Enrollment Rate (GER), and Kepulauan Bangka Belitung Province has the lowest GER at the college level in Indonesia. The March 2023 Susenas data indicates that four of the seven districts and cities in Kepulauan Bangka Belitung Province still have estimated GER values at the college level with insufficient precision. Therefore, to increase precision, indirect small area estimation (SAE) methods are required using auxiliary variables derived from Podes 2021. The research results show that SAE Hierarchical Bayes (HB) estimation using the beta distribution approach produces the best estimates compared to other methods for estimating GER at the college level.

Keywords: SDGs, GER at the college level, SAE HB, Province of Kepulauan Bangka Belitung.

1. INTRODUCTION

Quality education is one of the seventeen goals focused on by the Sustainable Development Goals (SDGs) to enhance human resource quality. Additionally, the 1945 Constitution states that one of the nation's objectives is to educate the life of the nation, where quality education is the primary means to achieve this goal. The importance of this program has made quality improvement one of the seven programs in the National Medium-Term Development Plan (NMTDP) 2020-2024.

One way to improve the quality of education is by increasing educational participation [24].The Gross Enrollment Rate (GER) is one of the indicators that can be used to show the level of educational participation. Based on the March 2023 Survei Sosial Ekonomi Nasional (Susenas), the college level has the lowest GER compared to other levels of education, at 31.45%. This figure has not yet reached the GER target for the college level in the 2020-2024 National Medium-Term Development Plan (NMTDP) of 37.63% [3]. It also falls short of the GER target for the college level in the SDGs for 2024, which is 38.32% under the business-as-usual scenario or 43.86% under the intervention scenario [2].

Based on provincial data, the Province of Kepulauan Bangka Belitung has the lowest GER at the college level in Indonesia, with a GER of only 18.19% [4]. Furthermore, this province, known as the "Bumi Serumpun Sebalai" has consistently held the lowest GER at the college level in

Indonesia since 2015. This indicates that the residents of Kepulauan Bangka Belitung are still not fully aware of the importance of education, particularly higher education. Therefore, the education system in the Province of Kepulauan Bangka Belitung still needs to be improved.

The low level of educational participation at the college level requires the regional government of the Province of Kepulauan Bangka Belitung to address this issue seriously. The local government must use precise data when formulating education policies to ensure they are targeted effectively. Data on small areas is essential for policy formulation, especially for improving education quality in a targeted manner [12].

Badan Pusat Statistik (BPS), as the provider of statistical data on GER at the college level through surveys such as Susenas, can only publish data at the national and provincial levels due to sample limitations [1]. Meanwhile, to address the issue of low educational participation at the college level, data at more granular levels, such as districts or cities, is needed. Direct estimation at these smaller levels can result in inadequate precision when there is sample insufficiency, making the analysis based on such conditions unreliable [23].

One solution to address sample insufficiency is to use the Small Area Estimation (SAE) method. The SAE method employs indirect estimation for small areas by incorporating auxiliary variables to enhance sample effectiveness and estimation efficiency [17]. A small area is defined as a segment of the population with the variable of interest, in this case, the GER at the college level [25].

Based on the availability of auxiliary variables, SAE models can be classified into two types: area-level models and unit-level models [17]. Area-level models are used when auxiliary variables are only available at the target area level, while unit-level models are employed when auxiliary variables are available down to the smallest sampling unit (ultimate sampling unit) [11]. In this study, an area-level model is used due to the limitations in the availability of auxiliary variables at the unit level.

Parameter estimation in the SAE method using area-level models can be performed with various techniques, such as Empirical Best Linear Unbiased Prediction (EBLUP), Empirical Bayes (EB), and Hierarchical Bayes (HB) [18]. The differences between these methods lie in their assumptions. The EBLUP method assumes that the data are normally distributed, whereas the EB and HB methods do not require the data to be normally distributed [17].

The EBLUP method, in addition to assuming that the data are normally distributed, has limitations when the number of observations is insufficient. The model may become less appropriate or unstable, for example producing zero or negative estimates of variance components [8]. In contrast, the HB method adopts more flexible assumptions and, according to [19], is considered more suitable for conditions with a limited number of observations. The ideal number of observations for SAE modeling using EBLUP is 30 [9]. Therefore, the Province of Kepulauan Bangka Belitung, which comprises only seven districts/cities, falls into the category of regions with fewer than 30 observations, a condition that may potentially introduce bias in the estimation results [15].

Therefore, the researcher is interested in implementing the SAE method with a Hierarchical Bayes approach to estimate the GER at the college level at the district/city level in the Province of Kepulauan Bangka Belitung for 2023, where there are only seven districts/cities. The results from the best method can be used as a basis for the government of the Province of Kepulauan Bangka Belitung to develop more targeted policies to improve the quality of higher education.

2. METHOD
2.1. Literature Review

Gross Enrollment Rate at the College Level

Gross Enrollment Rate at the college level is the ratio between the number of people currently pursuing higher education (regardless of their age) and the number of people in the ideal

age group for higher education, which is 19-23 years old [6]. A higher GER value indicates a higher level of participation in higher education in the region.

Empirical Best Linear Unbiased Predictor (EBLUP)

The EBLUP method is the most basic method used in SAE area-level models. Among the three estimation methods for area-level models, EBLUP is also the most commonly used. Developed a method based on the assumption of known variance components, known as the Best Linear Unbiased Prediction (BLUP) method [10]. BLUP estimation is based on a weighted average of direct and model-based estimates, optimizing the weights for the best prediction. Generally, the BLUP equation at the area level is as follows [15]:

$$\tilde{\theta}_i^H = x_i^T \tilde{\boldsymbol{\beta}} + \gamma_i(\hat{\theta}_i - x_i^T \tilde{\boldsymbol{\beta}}) = \gamma_i \hat{\theta}_i + (1 - \gamma_i) x_i^T \tilde{\boldsymbol{\beta}} \tag{2.1}$$

With $\hat{\theta}_i$ representing the direct estimate and $x_i^T \tilde{\boldsymbol{\beta}}$ representing the model estimate, where γ_i is the weight chosen with a value of $0 \leq \gamma_i \leq 1$, as given by the following formula:

$$\gamma_i = \frac{\sigma_v^2 b_i^2}{\psi_i + \sigma_v^2 b_i^2} \tag{2.2}$$

Based on equation (2.2), it can be seen that BLUP estimation depends on the variance component (σ_v^2), which is assumed to be known. In practice, however, the value of the variance component (σ_v^2) is not known and thus needs to be estimated. When the variance component (σ_v^2) is substituted with its estimate $(\hat{\sigma}_v^2)$, the EBLUP estimate is obtained, which can be written as follows [17]:

$$\hat{\theta}_i^H = \hat{\gamma}_i \hat{\theta}_i + (1 - \hat{\gamma}_i) x_i^T \hat{\boldsymbol{\beta}} \tag{2.3}$$

Hierarchical Bayes Beta

The beta distribution is suitable for modeling data within the range of 0-1 [22]. The SAE HB Beta model at the area level can be written as follows [14]:

i. Sampling model

$$\hat{\theta}_i | \theta_i \sim Beta\ (a_i, b_i), \qquad i = 1, \dots, m \tag{2.4}$$

θ_i in equation (2.4) represents a parameter with a range of values between 0 and 1, $\hat{\theta}_i$ is the estimator for the small area i, and $a_i = \theta_i k$ and $b_i = (1 - \theta_i)k$ are parameters of the beta distribution. Here, k is a constant assumed to follow a gamma distribution $k \sim Gamma(g_1, g_2)$.

ii. Linking model

$$logit(\theta_i) | \boldsymbol{\beta}, \sigma_v^2 \overset{iid}{\sim} N(x_i^T \boldsymbol{\beta}, \sigma_v^2), \qquad i = 1, \dots, m \tag{2.5}$$

Where $\boldsymbol{v_i} \sim N(0, \sigma_v^2)$ and $\sigma_v^2 \sim IG(c_1, c_2)$. In this case, IG stands for the Inverse Gamma distribution.

iii. The parameters $\boldsymbol{\beta}$ and σ_v^2 are assumed to have a flat prior and are independent of each other, where $\boldsymbol{\beta} \sim N(\mu_\beta, \sigma_\beta^2)$.

In Hierarchical Bayes inference for the parameters θ_i, $\boldsymbol{\beta}$, and σ_v^2 the process is conducted using Markov Chain Monte Carlo (MCMC) with the Gibbs Sampling algorithm, as implemented [14].

2.2. Data and Data Sources

This study uses secondary data obtained from the BPS through the March 2023 Survei Sosial Ekonomi Nasional (Susenas) for the variable of interest, GER at the college level, and data from the 2021 Potensi Desa (Podes) for auxiliary variables. In this study, there are 18 candidate auxiliary variables to be used, namely: Ratio of Senior High Schools per 10,000 people (X_1), Ratio of Vocational High Schools per 10,000 people (X_2), Ratio of Higher Education Institutions per 10,000 people (X_3), Ratio of healthcare facilities per 10,000 people (X_4), Ratio of healthcare personnel per

10,000 people (X_5), Percentage of villages with people's business credits (X_6), Percentage of villages with joint business groups (X_7), Percentage of villages with government banks (X_8), Percentage of villages with private banks (X_9), Percentage of villages with people's credit banks (X_{10}), Percentage of villages with minimarkets or supermarkets (X_{11}), Percentage of households using electricity (X_{12}), Percentage of villages with educational programs A/B/C (X_{13}), Percentage of issued poor certificates (X_{14}), Percentage of villages with urban status (X_{15}), Percentage of villages with public transportation (X_{16}), Percentage of villages with internet cafes (X_{17}), and Percentage of villages with Base Transceiver Stations (X_{18}).

2.3. Research Steps

The steps for estimating the GER at the college level in this study are outlined as follows:
1. Perform direct estimation of the GER at the college level at the district/city level based using R Studio with the survey package.
2. Prepare the candidate auxiliary variable data to be used.
3. Select the auxiliary variables to be used with a stepwise regression method.
4. Perform indirect estimation.
5. Evaluate the models to determine the best method for estimating the GER at the college level using the Relative Standard Error (RSE) value.
6. Map the estimation results of the GER at the college level based on the best estimation method.

3. MAIN RESULTS

From the direct estimation process of the GER at the college level based on the sampling design of the March 2023 Susenas, direct estimates and Relative Standard Error (RSE) values for each district/city in the Province of Kepulauan Bangka Belitung were obtained, as shown in Table 3.1 below.

Table 3.1. Descriptive statistics of direct estimates.

Descriptive Statistics	GER at the College Level	RSE (percent)
Minimum	0.08	16.20
Q1	0.11	20.82
Median	0.12	27.61
Mean	0.16	26.13
Q3	0.21	30.83
Maximum	0.29	35.82

Source: Susenas March 2023

Based on the maximum and minimum values of the GER at the college level shown in Table 3.1, it is observed that the data for the GER at the college level falls within the range of 0 to 1. Furthermore, for the direct estimation RSE values, it is noted that four out of seven districts/cities in Kepulauan Bangka Belitung have estimates with insufficient precision (RSE > 25%). These four districts/cities are West Bangka Regency, Central Bangka Regency, South Bangka Regency, and East Belitung Regency. This indicates that the direct statistical estimates of the GER at the college level should be used with caution due to their lack of precision [6]. The low precision in these four districts in the Province of Kepulauan Bangka Belitung is due to the insufficient sample size for direct estimation at the district level. Therefore, indirect estimation (SAE) is needed to achieve better

precision compared to direct estimation for estimating the GER at the college level in the Province of Kepulauan Bangka Belitung.

Indirect estimation begins with an area-level model using the EBLUP method. Based on the stepwise process between direct estimation and candidate auxiliary variables [20], three variables were found to be significant at the 5% significance level: Ratio of Vocational High Schools per 10,000 people (X_2), Ratio of healthcare personnel per 10,000 people (X_5), and the percentage of villages with urban status (X_{15}). These three variables will be used in estimating the GER at the college level at the district/city level in the Province of Kepulauan Bangka Belitung for 2023. From the EBLUP estimation process, it was found that only two variables were significant in the model, as presented in Table 3.2 below.

Table 3.2. Best SAE EBLUP model parameter estimates.

Variable	$\hat{\beta}$	*Standard Error*	$p-value$
(intercept)	-0.1074	0.0580	0.0642
X_5	0.0154	0.0039	0.0001*
X_{15}	0.0018	0.0007	0.0118*
variance of the random effect area ($\hat{\sigma}_v^2$) = 0			

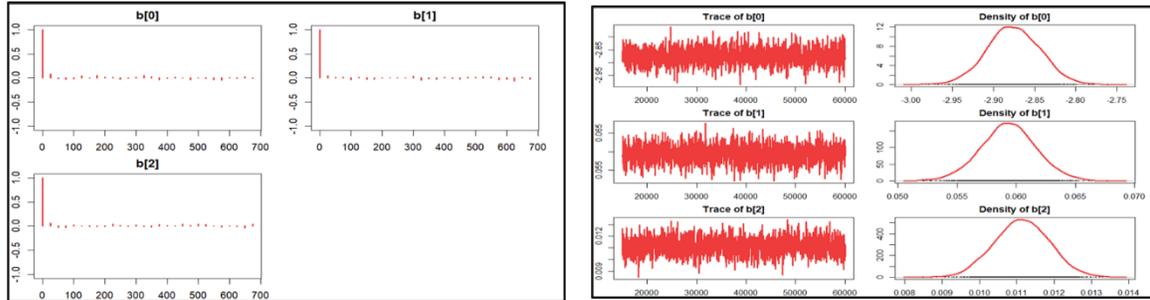*Source: Susenas March 2023 and Podes 2021*
Note: (*) significant at $\alpha = 5\%$

Based on Table 3.2, the estimated beta ($\hat{\boldsymbol{\beta}}$) from the EBLUP model shows that statistically, all variables have a significant effect, except for the model intercept. The estimation results from the EBLUP model also indicate that the estimated variance of the random effect area ($\hat{\sigma}_v^2$) is zero. Since $\hat{\sigma}_v^2 = 0$, it results in the random effect area (v_i) also being zero, which means the assumptions of EBLUP are not met. This is indicated by the weakness of the EBLUP method in estimating the GER at the college level in the Province of Kepulauan Bangka Belitung, which has only seven districts/cities. This result is consistent with [14], who noted that one weakness of the EBLUP method is that it can produce zero estimates for the variance components. The Hierarchical Bayes model is better suited for cases where $\hat{\sigma}_v^2 = 0$ [21]. Therefore, the Hierarchical Bayes model will be used to estimate the GER at the college level in the Province of Kepulauan Bangka Belitung.

It is known that the direct estimation of the GER at the college level has values ranging from 0 to 1. Therefore, the appropriate SAE HB method to use is the SAE HB with a beta distribution. The beta distribution is suitable for modeling data within the range of 0-1 [22]. Thus, the indirect estimation (SAE HB) in this study will use the Beta distribution approach, which is considered more appropriate than other distributions for estimating the GER at the college level in the Province of Kepulauan Bangka Belitung.

Similarly to the EBLUP model estimation process, the estimation of the Hierarchical Bayes Beta model also begins with selecting the auxiliary variables to be included in the model using a stepwise regression process. In the HB Beta method, the candidate auxiliary variables are regressed with the logit form of the direct estimates [16]. From the stepwise process between the logit of the direct estimates and the candidate auxiliary variables, three variables were found to be significant at the 5% significance level: Ratio of Vocational High Schools per 10,000 people (X_2), Ratio of healthcare personnel per 10,000 people (X_5), and the percentage of villages with urban status (X_{15}). Therefore, these three variables from the stepwise process can be used to estimate the GER at the college level in the Province of Kepulauan Bangka Belitung using the SAE HB Beta method.

Parameter estimates from the HB Beta model can be obtained from a converged MCMC algorithm. Using a total of 60,000 iterations, with a burn-in of 15,000, and thinning and updating both set to 25, the final number of converged samples was 1,800. From the MCMC results, it was found that the variable X_2 was not significant and therefore was excluded from the estimation process.

Source: Susenas March 2023 and Podes 2021
**Figure 3.1**. *diagnostic plot* for all parameters.

Based on the diagnostic plots in Figure 3.1, each of which indicates that the MCMC algorithm has converged, the parameter estimates of the HB Beta model can be seen in Table 3.3 below. It is observed that all parameters in the HB Beta model are significant. This is because the credible intervals from 2.5% to 97.5% for each parameter do not include zero.

Table 3.3. Best SAE HB model parameter estimates.

Coefficient Estimation Parameter	Mean	Standard Deviation	2.5%	97.5%
b_0	-2.8755	0.0320	-2.9379	-2.8131
b_1	0.0594	0.0023	0.0547	0.0639
b_2	0.0111	0.0007	0.0097	0.0126

Source: Susenas March 2023 and Podes 2021

After obtaining the estimation results of the GER at the college level from several models, the next step is to evaluate these model estimates based on their precision to determine the best estimation method for the GER in the Province of Kepulauan Bangka Belitung. In this study, the Relative Standard Error (RSE) value is used as a measure to determine the precision of the estimation results. The smaller the RSE value of the estimation model, the more precise the estimate is. Therefore, the best model will be selected based on the lowest RSE value.
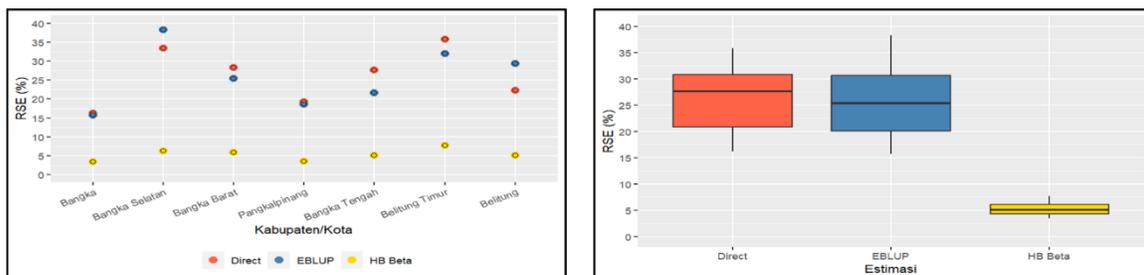

(a)   (b)

Source: Susenas March 2023 and Podes 2021
Figure 3.2. (a) Point graph of RSE value and (b) *Boxplot* of RSE value for each estimation model.

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Based on Figure 3.2, it can be seen that the EBLUP estimation results do not perform well based on the RSE values. This is consistent with findings from [1] and [13], which show that when the observations are limited, EBLUP estimation does not perform better than direct estimation, as indicated by the higher RSE values for EBLUP compared to direct estimates. In contrast, the HB Beta estimation method produces the lowest RSE values compared to other methods across all districts/cities in the Province of Kepulauan Bangka Belitung.

The same observation is evident from the width of the boxplot for the RSE values of the HB Beta estimation, which has the narrowest and most distinct box compared to the other boxplots. This indicates that the HB Beta estimation is more precise, as it is located well below the boxplots of the other estimations. Therefore, the SAE HB Beta estimation is the best method for estimating the GER at the college level in the Province of Kepulauan Bangka Belitung for the year 2023.

To examine the geographic distribution of the GER at the college level in the Province of Kepulauan Bangka Belitung for 2023, a thematic map is presented below, with the GER categorized into three groups based on natural breaks. The Province of Kepulauan Bangka Belitung is an archipelagic province, consisting of two major islands: Bangka Island and Belitung Island. According to Figure 3.3, it can be seen that the regions in the Province of Kepulauan Bangka Belitung with the highest GER, ranging from 14.10% to 28.90%, are Pangkalpinang City and Bangka Regency on Bangka Island. These two areas are directly adjacent, suggesting a spatial factor where regions closer to the provincial capital tend to have higher GERs.
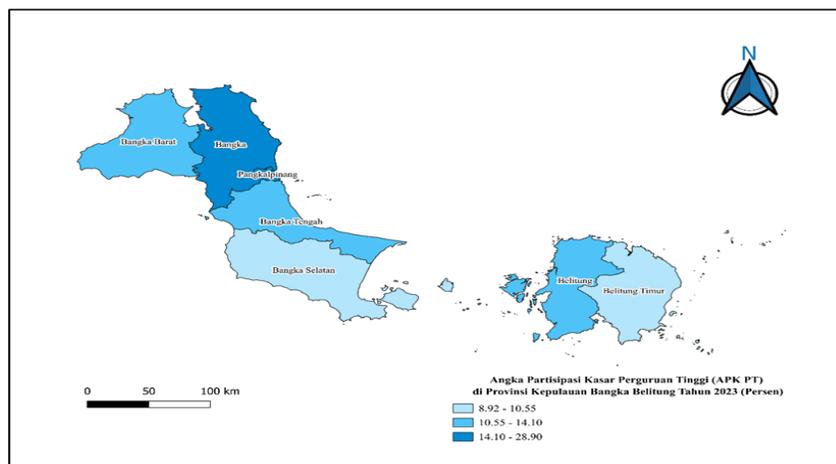
Source: Susenas March 2023 and Podes 2021

Figure 3.3. Thematic map of GER at the College Level the Province of Kepulauan Bangka Belitung in 2023 on the SAE HB Beta method.

Similarly, Belitung Timur Regency has the lowest GER at the college level because it is the furthest from the provincial capital. In addition to Belitung Timur Regency, Bangka Selatan Regency also falls into the category of the lowest GER, ranging from 8.92% to 10.55%. The remaining districts—Bangka Barat, Bangka Tengah, and Belitung have GERs ranging from 10.55% to 14.10%.

Based on the regional map, it is evident that Bangka Selatan Regency and Belitung Timur Regency have areas with small islands, requiring residents in these regions to rely on sea or air transportation to access the city center, which increases transportation costs. This is consistent with [1], who state that one of the factors contributing to the low participation in higher education is the accessibility and high travel costs.

Additionally, the province known as Bumi Serumpun Sebalai is also one of the largest tin producers in Indonesia. In 2023, the mining and quarrying sector was among the top five sectors contributing the most to the Gross Regional Domestic Product (GRDP) of the Province of Kepulauan Bangka Belitung, with Bangka Selatan Regency and Belitung Timur Regency being the largest contributors to this sector [7]. The high contribution of the mining sector leads many residents to prefer working directly in mining rather than pursuing higher education. This is also consistent with the fact that the proportion of the population working in the mining and quarrying sector is larger compared to other sectors, at 28.85% [4]. Therefore, it is understandable that these two districts have a lower GER at the college level compared to other areas in the Province of Kepulauan Bangka Belitung.

In order to achieve the goal of improving education quality outlined in the fourth SDG and the National Medium-Term Development Plan (RPJMN) 2020-2024, the Indonesian government, particularly regional governments, can utilize the results from the Hierarchical Bayes Beta (HB Beta) estimation to plan higher education policies. As a province that has consistently ranked lowest in the GER at the college level in Indonesia, quality education policies should be implemented and prioritized for the Province of Kepulauan Bangka Belitung, especially for the regional governments of Belitung Timur Regency and Bangka Selatan Regency, which have a GER ranging from 8.92% to 10.55%. These figures are significantly below the national average of 31.45% and still far from the SDG and RPJMN targets of over 35%.

4. CONCLUSION

Based on the research results outlined previously, the direct estimation of the GER at the college level for districts/citys in the Province of Kepulauan Bangka Belitung in 2023 ranges from 0 to 1, with generally high RSE values above 25%, indicating that the direct estimates lack adequate precision. Therefore, an indirect estimation of the GER at the college level for districts/citys, obtained using the SAE HB method with a Beta distribution approach and auxiliary variables from Podes 2021, is needed. The best estimation method for the GER at the college level in the Province of Kepulauan Bangka Belitung in 2023, which has only seven districts/citys, is the SAE HB method with the Beta distribution approach. This is evidenced by RSE values for all districts/citys being below 25%. Furthermore, the mapping results show that the area of Pangkalpinang City, as the capital of the Province of Kepulauan Bangka Belitung, and its surrounding areas will have higher GER at the college level compared to regions farther from the provincial capital.

REFERENCES

[1]   Afifah, U. N., & Faradis, R., 2020. Estimation Gross Enrolment Rate of Higher Education in Bangka Belitung Island using Small Area Estimation. Indonesian Journal of Development Studies, 1(1), 26–30. https://doi.org/10.12962/j29649714.v1i1.8280
[2]   Bappenas, 2020a. Peta Jalan SDGs Indonesia Menuju 2030. Jakarta: Kementerian Perencanaan Pembangunan Nasional.
[3]   Bappenas, 2020b. Rencana Pembangunan Jangka Menengah Nasional 2020-2024. Jakarta: Kementerian Perencanaan Pembangunan Nasional.
[4]   BPS, 2023a. Angka Partisipasi Kasar (APK) Perguruan Tinggi (PT) Menurut Provinsi, 2021-2023. https://www.bps.go.id/id/statistics-table/2/MTQ0MyMy/angka-partisipasi-kasar--apk--perguruan-tinggi--pt--menurut-provinsi.html

[5] BPS, 2023b. Keadaan Angkatan Kerja Provinsi Kepulauan Bangka Belitung Tahun 2022. www.freepik.com

[6] BPS, 2023c. Statistik Pendidikan 2023. In Jakarta: BPS.

[7] BPS, 2024. Provinsi Kepulauan Bangka Belitung Dalam Angka 2024 (Vol. 23).

[8] Chen, S., Lahiri, P., & Rao, J. N. K., 2025. Mean squared prediction error estimators of the empirical best linear unbiased predictor of a small area mean under a semi-parametric Fay-Herriot model. Survey Methodology.

[9] Datta, G. S., & Lahiri, P., 2000. A unified measure of uncertainty of estimated best linear unbiased predictors in small area estimation problems. Statistica Sinica, 613–627.

[10] Henderson, C. R., 1975. Best linear unbiased estimation and prediction under a selection model. Biometrics, 423–447. https://doi.org/10.2307/2529430

[11] Ikhsan, E., Ratu, N. Y., & Nurizza, W. A., 2019. Estimasi Small Area Estimation Angka Partisipasi Kasar di Perguruan Tinggi Pulau Kalimantan Tahun 2018. Prosiding Seminar Nasional Matematika Dan Statistika, 82–88.

[12] Istiqomah, N. P., & Wulansari, I. Y., 2022. Estimasi Angka Partisipasi Kasar Perguruan Tinggi Level Kabupaten/Kota di Pulau Kalimantan Tahun 2020 dengan Small Area Estimation Hierarchical Bayes Beta-Logistic.

[13] Kurniawan, R., Arifatin, D., Noviani, A., & Fadhlullah, F., 2019. Evaluasi Pendugaan Angka Partisipasi Kasar Perguruan Tinggi Tahun 2018 Dengan Small Area Estiomation Benchmarking. Seminar Nasional Official Statistics, 2019(1), 67–73. https://doi.org/10.34123/semnasoffstat.v2019i1.86

[14] Liu, B., 2009. Hierarchical Bayes estimation and empirical best prediction of small-area proportions. University of Maryland, College Park. http://hdl.handle.net/1903/9149

[15] Marcis, L., Pagliarella, M. C., & Salvatore, R., 2024. How the sampling variances affect the linear predictor of the Fay-Herriot model. METRON, 82, 109–130. https://api.semanticscholar.org/CorpusID:269147493

[16] Moretti, A., 2023. Estimation of small area proportions under a bivariate logistic mixed model. Quality & Quantity, 57(4), 3663–3684. https://doi.org/10.1007/s11135-022-01530-6

[17] Pratama, A. Y., 2024. Application of Small Area Estimation for Estimation of Sub-District Level Poverty in Bengkulu Province: Comparison of Empirical Best Linear Unbiased Prediction (EBLUP) and Hierarchical Bayesian (HB) Methods. Journal of Statistics and Data Science, 3(1), 1–12.

[18] Rao, J. N. K., & Molina, I., 2015. SMALL AREA ESTIMATION (M. P. Couper, G. Kalton, J. N. K. Rao, N. Schwarz, & C. Skinner, Eds.; 2nd ed.). John Wiley & Sons, Inc., Hoboken, New Jersey. https://doi.org/10.1002/9781118735855

[19] Raudenbush, S. W., & Bryk, A. S., 2002. Hierarchical linear models: Applications and data analysis methods: Vol. 2nd edition. sage.

[20] Ren, P., Li, J., Erciulescu, A. L., Krenzke, T., & Mohadjer, L., 2022. A variable selection method for small area estimation modeling of the proficiency of adult competency. Stats, 5(3), 689–713

[21] Saadi, R., & Ubaidillah, A., 2022. Pendugaan Area Kecil Angka Partisipasi Kasar Perguruan Tinggi di Indonesia Menggunakan Model Subarea Twofold dengan Pendekatan Hierarchical Bayes. Seminar Nasional Official Statistics, 2022(1), 123–136. https://doi.org/10.34123/semnasoffstat.v2022i1.1217

[22] Souza, D. F., & Moura, F. A. S., 2016. Multivariate Beta Regression with Application in Small Area Estimation. Journal of Official Statistics, 32(3), 747–768. https://doi.org/10.1515/jos-2016-0038

[23] Ubaidillah, A., 2014. Small Area Estimation dengan Pendekatan Hierarchical Bayesian Neural Network untuk Pemetaan Kemiskinan di Kota Jambi.

[24] Utami, E. D., & Wicaksono, F., 2019. Determinan Partisipasi Sekolah di Indonesia: Analisis Data Susenas Tahun 2017. Jurnal Ilmiah Widya Non-Eksakta Vol, 1, 68–73.

[25] Yanuar, F., Fajriyah, R., & Devianto, D., 2021. Small Area Estimation Method with Empirical Bayes Based on Beta Binomial Model in Generated Data. Media Statistika, 14(1), 1–9.