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First Zagreb, Gutman, and Wiener Index of the Non-Coprime Graph for Dihedral Group

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Abstract

This study investigates the structure of non-coprime graphs derived from the dihedral groups D_{2n} using approaches from graph theory and group theory. In molecular representation and pure mathematics, graphs serve as tool to illustrate the internal structure of systems through topological indices. A non-coprime graph is a graph formed by connecting two distinct vertices if the greatest common divisor (GCD) of their orders is not equal to one, meaning the two elements are not coprime. The primary focus of this research is to derive general formulas for the first Zagreb index, the Gutman index, and the Wiener index of non-coprime graphs constructed from the dihedral group D_{2n}. For the first Zagreb index, the case $n = p^k$ are where k is a positive integer. Meanwhile, for the Gutman and Wiener indices, the study focuses on the case $n = 2^k$, with k is a positive integer. This research produces explicit formulas for the first Zagreb index, Gutman index, and Wiener index of the corresponding non-coprime graphs.

Keywords: non-coprime graph, dihedral group, first Zagreb index, Gutman index, Wiener index

1. INTRODUCTION AND PRELIMINARIES

Graph theory is one of the mathematical frameworks that can be applied to solve various real-life problems, including those in chemistry, games, transportation, and commerce [4]. Furthermore, in the context of pure mathematics, graphs are often used as a representation tool to describe the structure of a group [10]. A further application of graph theory in mathematics is the determination of topological indices. A topological index is a numerical metric used to measure the characteristics of a graph, such as distance, connectivity, and other structural properties [12].

Research on topological indices applied to graphs associated with certain groups has been widely conducted. For instance, [5] studied the Harary index of the coprime graph of a dihedral group, while

[7] investigated the Sombor index of the non-coprime graph of a quaternion group. In another study, [8] identified structural patterns of non-coprime graphs of dihedral groups for the cases $n = p^k$ and $n = 2^k$ where p is a prime number and k is a positive integer. These results were later utilized in [11], who derived a general formula for the Sombor index of non-coprime graphs of dihedral groups. To date, research focusing on deriving general formulas for topological indices applied to non-coprime graphs of dihedral groups remains limited to these works.

Therefore, this study aims to derive general formulas for topological indices applied to the non-coprime graph of a dihedral group. The topological indices considered in this research are the first Zagreb index, the Gutman index, and the Wiener index. Specifically, for the first Zagreb index, the cases $= p^k$ and $n = 2^k$ where p is a prime number and k is a positive integer. Meanwhile, for the Gutman and Wiener indices, the analysis is restricted to the case $n = 2^k$, with k being a positive integer.

2. RESEARCH METHODOLOGY

This research employs a deductive method to explore new insights into algebraic structures that have been previously studied. The approach begins by analyzing algebraic structures in specific cases to identify particular patterns. Based on the patterns found, this study then formulates hypotheses applicable to more general cases.

In the context of the non-coprime graph, this study focuses on the form p^k where p is a prime and k is a natural number forming the basis for graph construction.

To begin the exploration, the definition of the dihedral group is given forms the primary algebraic framework for constructing the non-coprime graph in this research.

Definition 1.1. [3] *A group D_{2n} is said to be a dihedral group of order $2n$, where $n \geq 3$ and $n \in \mathbb{N}$, if it is generated by the elements $a, b \in D_{2n}$, with the group presentation*

$$D_{2n} = \langle a, b | a^n = e, b^2 = e, bab^{-1} = a^{-1} \rangle$$

Having the dihedral group, the non-coprime graph is presented as central concept that allows us to represent the relationships between elements of the group based on their orders.

Definition 1.2. [6] *Let G be a finite group. Non-coprime graph of G, denoted by $\overline{\Gamma_G}$ is a graph with its vertices consist of $G - \{e\}$ and two different vertices u, v are said to be adjacent if $(|u|, |v|) \neq 1$.*

In order to quantify the structural properties of the non-coprime graphs, we utilize specific topological indices. The first of these is the first Zagreb index, which is defined as follows.

Definition 3.3.[2] *Let Γ be a simple graph. The first Zagreb index of Γ denoted by $M_1(\Gamma)$, is:*

$$M_1(\Gamma) = \sum_{u \in V(\Gamma)} (deg(u))^2$$

where $deg(u)$ is the degree of vertex u

Besides degree-based measures, another important topological index used in this study is the Gutman index, which captures information related to the degrees and distances between vertices.

Definition 3.4.[1] *Let Γ be a simple graph. The Gutman index of Γ denoted by $Gut(\Gamma)$, is:*

$$Gut(\Gamma) = \sum_{u,v \in V(\Gamma)} deg(u)\, deg(v)\, d(u,v)$$

where $d(u,v)$ is the distance between vertices u and v

Another key distance-based topological index is the Wiener index, which measures the sum of distances between all pairs of vertices in a graph. Its formal definition is given below.

Definition 3.5. [9] *Let Γ be a simple graph. The Wiener index Γ denoted by $W(\Gamma)$, is:*

$$W(\Gamma) = \sum_{u,v \in V(\Gamma)} d(u,v)$$

Before proceeding to the computation of the indices, we first establish important structural properties of non-coprime graphs associated with dihedral groups, starting with the following lemma.

Lemma 3.6. [8] *Let D_{2n} be the dihedral group of order $2n$. If $n = 2^m$ for some $m \in \mathbb{N}$, then $\overline{\Gamma D_{2n}}$ is a complete graph $K_{2^{m+1}-1}$.*

Proof. Since $n = 2^m$, the order of D_{2n} is $2n = 2^{m+1}\}$. Hence, the order of every non-identity element of D_{2n} must be divided by 2. For any non-identity $u, v \in D_{2n}$, we have $\gcd(|u|,|v|) > 1$. Therefore, every pair of distinct vertices is adjacent, and thus $\overline{\Gamma D_{2n}}$ is a complete graph with $2^{m+1} - 1$ vertex.

To complement the previous lemma, the next result characterizes the vertex partitioning in the non-coprime graph when the group order is a multiple of an odd prime.

Lemma 3.7. [8] *Let D_{2n} be the dihedral group of order $2n$. If $n = p^m$ for some $m \in N$ and p is an odd prime, then $\overline{\Gamma D_{2n}}$ can be partitioned into disjoint union of two complete graphs $K_{p^m-1} \cup K_{p^m}$.*

Proof. Let split $D_{2n} - \{e\}$ into two disjoint sets $G_1 = \{a, a^2, \dots, a^{p^m-1}\}$ and $G_2 = \{b, a^2 b, \dots, a^{p^m-1}\mathrm{b}\}$. We have p divides the order of any $u \in G_1$ and 2 divides the order of any $v \in G_2$ since u and v are rotation and reflection elements, respectively. So we have every two elements in G_1 are neigbors for $i = 1,2$. Since p is odd prime then for any $u \in G_1$ and $v \in G_2$, we have $(|u|,|v|) = 1$. Hence u and v cannot be neighbors. Therefore $\overline{\Gamma D_{2n}}$ can be partitioned into two disjoint complete graphs.

3. MAIN RESULTS

Theorem 2.1. *Let $\overline{\Gamma D_{2n}}$ be the non-coprime graph of the dihedral group D_{2n}. If $n = p^k$ with p is a prime number and , $p \neq 2$ where $k \in \mathbb{N}$, then the first Zagreb index of the non-coprime graph of the dihedral group is:*

$$M_1\left(\overline{\Gamma D_{2(p^k)}}\right) = 2(p^k)^3 - 7(p^k)^2 + 9(p^k) - 4$$

Proof. Let $\overline{\Gamma D_{2n}}$ be the non-coprime graph of the dihedral group D_{2n}. If $n = p^k$ with p is a prime number and , $p \neq 2$ where $k \in \mathbb{N}$, the non-coprime graph obtained consists of two vertex partitions,

$V_1(\overline{\Gamma D_{2n}}) = \{a, a^2, \dots, a^{n-1}\}$ and $V_2(\overline{\Gamma D_{2n}}) = \{b, ab, a^2b, \dots, a^{n-1}b\}$. Thus, the formula for the first Zagreb index of the non-coprime graph of the dihedral group is as follows:

$$M_1\left(\overline{\Gamma D_{2(2^k)}}\right) = \sum_{u \in V_1(\overline{\Gamma D_{2n}})} (\deg(u))^2 + \sum_{u \in V_2(\overline{\Gamma D_{2n}})} (\deg(u))^2$$

Based on Lemma 3.7, $\overline{\Gamma D_{2p^k}} = K_{p^k-1} \cup K_{p^k}$. Hence, for $u \in V_1(\overline{\Gamma D_{2n}})$, $\deg(u) = (n-2)$ and for $v \in V_2(\overline{\Gamma D_{2n}})$, $\deg(v) = (n-1)$. The number of vertices in $V_1(\overline{\Gamma D_{2n}}) = (n-1)$ and the number of vertices in $V_2(\overline{\Gamma D_{2n}}) = n$. So, it is obtained that:

$$\begin{aligned}
M_1(\overline{\Gamma D_{2n}}) &= (n-1)(n-2)^2 + n(n-1)^2 \\
&= (n-1)(n^2-4n+4) + n(n^2-2n+1) \\
&= n^3 - 4n^2 + 4n - n^2 + 4n - 4 + n^3 - 2n^2 + n \\
&= 2n^3 - 7n^2 + 9n - 4 \\
&= 2(p^k)^3 - 7(p^k)^2 + 9(p^k) - 4.
\end{aligned}$$

Theorem 2.2. *Let $\overline{\Gamma D_{2n}}$ be the non-coprime graph of the dihedral group D_{2n}. If $n = 2^k$, where $k \in \mathbb{N}$, then the first Zagreb index of the non-coprime graph of the dihedral group is:*

$$M_1(\overline{\Gamma D_{2n}}) = 8(2^k)^3 - 20(2^k)^2 + 16(2^k) - 4$$

Proof. Let $\overline{\Gamma D_{2n}}$ be the non-coprime grapsh of a dihedral group. If $n = 2^k$, where $k \in \mathbb{N}$. Based on Lemma 3.6, $\overline{\Gamma D_{2(2^k)}} = K_{2^{k+1}-1}$. Hence, for $u \in V(\overline{\Gamma D_{2n}})$ $\deg(u) = (2n-2)$. The number of vertices u is $(2n-1)$. So, it is obtained that:

$$\begin{aligned}
M_1(\overline{\Gamma D_{2n}}) &= \sum_{u \in V(\overline{\Gamma D_{2n}})} (\deg(u))^2 \\
&= (2n-1)(2n-2)^2 \\
&= (2n-1)(4n^2-8n+4) \\
&= 8n^3 - 16n^2 + 8n - 4n^2 + 8n - 4 \\
&= 8n^3 - 20n^2 + 16n - 4 \\
&= 8(2^k)^3 - 20(2^k)^2 + 16(2^k) - 4.
\end{aligned}$$

Theorem 2.3. *Let $\overline{\Gamma D_{2n}}$ be the non-coprime graph of the dihedral group D_{2n}. If $n = 2^k$, where $k \in \mathbb{N}$, then the Gutman index of the non-coprime graph of the dihedral group is:*

$$Gut\left(\overline{\Gamma D_{2(2^k)}}\right) = 8(2^k)^4 - 28(2^k)^3 + 36(2^k)^2 - 20(2^k) + 4$$

Proof. Let $\overline{\Gamma D_{2n}}$ be the non-coprime graph of the dihedral group D_{2n}. If $n = 2^k$, where $k \in \mathbb{N}$. Based on Lemma 3.6, $\overline{\Gamma D_{2(2^k)}} = K_{2^{k+1}-1}$. Hence, for $u, v \in V(\overline{\Gamma D_{2n}})$, $\deg(u) = \deg(v) = (2n-2)$ and $d(u,v) = 1$. The number of vertices is $2n-1$, so the number of pairs of vertices is $C_2^{2n-1} = 2n^2 - 3n + 1$. So, it is obtained that:

$$\begin{aligned}
Gut(\overline{\Gamma D_{2n}}) &= \sum_{u,v \in V(\overline{\Gamma D_{2n}})} deg(u)\, deg(v)\, d(u,v) \\
&= (2n^2-3n+1)(2n-2)(2n-2)(1) \\
&= (4n^2-8n+4)(2n^2-3n+1) \\
&= 8n^4 - 12n^3 + 4n^2 - 16n^3 + 24n^2 - 8n + 8n^2 - 12n + 4 \\
&= 8n^4 - 28n^3 + 36n^2 - 20n + 4 \\
&= 8(2^k)^4 - 28\left(2^k\right)^3 + 36\left(2^k\right)^2 - 20(2^k) + 4.
\end{aligned}$$

Theorem 2.4. *Let* $\overline{\Gamma D_{2n}}$ *be the non-coprime graph of the dihedral group* D_{2n}*. If* $n = 2^k$, *where* $k \in \mathbb{N}$*, then the Wiener index of the non-coprime graph of the dihedral group is:*

$$W\left(\overline{\Gamma D_{2(2^k)}}\right) = 2(2^k)^2 - 3(2^k) + 1$$

Proof. Let $\overline{\Gamma D_{2n}}$ be the non-coprime graph of the dihedral group D_{2n}. If $n = 2^k$, where $k \in \mathbb{N}$. Based on Lemma 3.6, $\overline{\Gamma D_{2(2^k)}} = K_{2^{k+1}-1}$. Hence, for $u, v \in V(\overline{\Gamma D_{2n}})$, $d(u,v) = 1$. The number of vertex pairs between u □□□ v is $2n^2 - 3n + 1$, $\forall\, u \in V(G)$. So, it is obtained that:

$$\begin{aligned} W\left(\overline{\Gamma D_{2(2^k)}}\right) &= \sum_{u,v \in V(G)} d(u,v) \\ &= (2n^2 - 3n + 1)(1) \\ &= 2n^2 - 3n + 1 \\ &= 2(2^k)^2 - 3(2^k) + 1. \end{aligned}$$

3. CONCLUSION

From the results above, the first Zagreb index for $n = p^k$ with p is a prime number and , $p \neq 2$ where $k \in \mathbb{N}$ is

$$M_1\left(\overline{\Gamma D_{2(p^k)}}\right) = 2(p^k)^3 - 7(p^k)^2 + 9(p^k) - 4.$$

This result shows that the first Zagreb index increases cubically with respect to the order of the group. Therefore, as the number of vertices in the non-coprime graph increases, the contribution of vertex degrees also grows significantly, indicating a denser graph structure and stronger connectivity among vertices.

Meanwhile, the first Zagreb index, Gutman index, and Wiener index for $n = 2^k$ where where k $\in \mathbb{N}$ are respectively

$$\begin{aligned} M_1\left(\overline{\Gamma D_{2(2^k)}}\right) &= 8(2^k)^3 - 20(2^k)^2 + 16(2^k) - 4 \\ Gut\left(\overline{\Gamma D_{2(2^k)}}\right) &= 8(2^k)^4 - 28(2^k)^3 + 36(2^k)^2 - 20(2^k) + 4 \\ W\left(\overline{\Gamma D_{2(2^k)}}\right) &= 2(2^k)^2 - 3(2^k) + 1 \end{aligned}$$

These results indicate that the Gutman index grows faster than the first Zagreb index and Wiener index because it depends simultaneously on both vertex degrees and distances between vertices. This implies that the non-coprime graph of the dihedral group becomes increasingly complex as the group order increases. Furthermore, the quadratic growth of the Wiener index shows that the overall distances between vertices increase in a more controlled manner compared to the rapid growth of degree-based indices.

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