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Insurance Portfolio Valuation under Market Jumps: A Framework Combining Kou Jump-Diffusion Forecasting and Dynamic CPPI Hedging

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Abstract

This study develops an integrated valuation framework for insurance portfolios under discontinuous market conditions. The framework combines asset-price forecasting based on the Kou jump-diffusion model with Dynamic Constant Proportion Portfolio Insurance (D-CPPI) to support portfolio protection and capital guarantee management. The Kou jump-diffusion model is employed to capture asymmetric jumps and extreme market movements, while D-CPPI dynamically allocates wealth between risky and risk-free assets. To address gap risk, a volatility-adjusted risk multiplier with a non-negativity constraint is incorporated into the portfolio insurance mechanism. The proposed framework is evaluated using S&P 500 index data from 2006–2010 at daily, weekly, and monthly observation frequencies. The forecasting results demonstrate satisfactory predictive performance, with Mean Absolute Percentage Error (MAPE) values of 4.76%, 5.04%, and 5.36% for the daily, weekly, and monthly models, respectively. Portfolio simulations indicate that arbitrarily selected multipliers can lead to substantial losses and floor violations, whereas the constrained volatility-adjusted multiplier maintains portfolio values above the guaranteed floor across all observation frequencies and effectively mitigates gap risk. These findings suggest that integrating Kou jump-diffusion forecasting with D-CPPI provides an effective approach for insurance portfolio valuation under volatile market conditions. The proposed framework contributes to the literature by jointly addressing asset-price jumps, dynamic portfolio allocation, and gap-risk mitigation within a unified insurance portfolio management framework.

Keywords: Insurance Portfolio Valuation, Kou Jump-Diffusion Model, Dynamic CPPI, Gap Risk Mitigation, Volatility-Adjusted Multiplier.

1. INTRODUCTION

Insurance premiums are invested to build portfolios that generate returns while ensuring the availability of funds to meet future claims and policyholder obligations. Consequently, insurers must manage investment portfolios that simultaneously pursue growth and preserve a guaranteed minimum value. This task becomes increasingly challenging in financial markets characterized by high volatility, sudden price changes, and extreme events [4]. Under such conditions, traditional portfolio valuation and risk management techniques may fail to adequately capture the risks faced by insurance products with embedded guarantees.

Accurate modeling and forecasting of asset price dynamics are fundamental to insurance portfolio valuation. Conventional diffusion-based models, such as Geometric Brownian Motion (GBM), assume continuous asset-price trajectories and therefore cannot fully represent the abrupt market movements frequently observed in practice. To address this limitation, jump-diffusion models incorporate discontinuous price changes into the asset-price process. Among these models, the Kou jump-diffusion model has attracted considerable attention because it employs an asymmetric double-exponential jump distribution, enabling it to capture both heavy-tailed behavior and asymmetric market shocks more effectively than standard diffusion models [5], [11]. Compared with the Merton jump-diffusion model, the Kou specification accommodates asymmetric upward and downward jumps, making it particularly suitable for representing skewed return distributions commonly observed in financial markets. Empirical studies have shown that jump-diffusion models often provide superior performance in describing asset-return dynamics during periods of financial stress [10]. In the context of insurance portfolio valuation, more accurate representation of future asset-price dynamics can improve portfolio allocation decisions and enhance the effectiveness of portfolio insurance strategies.

While accurate forecasting is essential, effective risk-control mechanisms are equally important for maintaining portfolio guarantees and managing downside risk. Constant Proportion Portfolio Insurance (CPPI) is one of the most widely used portfolio insurance strategies because it dynamically allocates wealth between risky and riskless assets while maintaining a predetermined protection floor. Dynamic CPPI (D-CPPI) extends this framework by adjusting the risk multiplier according to changing market conditions, thereby improving flexibility and responsiveness to market uncertainty [14]. However, despite its advantages, D-CPPI remains vulnerable to gap risk arising from sudden market jumps and discrete portfolio rebalancing, which may cause the portfolio value to breach the guaranteed floor [6].

While jump-diffusion models have been extensively studied for asset-price forecasting and CPPI-type strategies have been widely applied for portfolio protection, studies integrating jump-diffusion forecasting and CPPI-based portfolio insurance within a unified insurance portfolio valuation framework remain limited. Several studies have proposed methods to mitigate gap risk, including volatility-adjusted multipliers, analytically calibrated allocation rules, and option-based hedging strategies [10], [13]. Nevertheless, the existing literature largely examines forecasting models and portfolio insurance mechanisms separately. Limited attention has been devoted to integrating jump-aware forecasting, dynamic portfolio insurance allocation, and gap-risk mitigation within a unified insurance portfolio valuation framework. Furthermore, empirical evidence regarding the effectiveness of such an integrated approach across different observation frequencies remains scarce.

Motivated by these research gaps, this study develops an integrated insurance portfolio valuation framework that combines Kou jump-diffusion forecasting with Dynamic Constant Proportion Portfolio Insurance (D-CPPI) hedging. The proposed framework employs the Kou model to forecast future asset-price dynamics, uses D-CPPI to manage portfolio exposure, and incorporates a volatility-adjusted multiplier with a non-negativity constraint to mitigate gap risk and preserve portfolio guarantees. Using S&P 500 data at daily, weekly, and monthly frequencies, the study evaluates the ability of the proposed framework to maintain capital protection while preserving portfolio performance under varying market conditions. Unlike previous studies that examine forecasting and portfolio insurance separately, the proposed framework integrates jump-sensitive forecasting, volatility-adjusted portfolio insurance, and gap-risk mitigation within a unified valuation framework. The findings contribute to the literature by providing empirical evidence that jump-sensitive forecasting and volatility-adjusted D-CPPI allocation can be jointly employed to support insurance portfolio valuation while preserving capital guarantees and mitigating gap risk under volatile market conditions.

2. KOU JUMP FORECASTING MODEL

Modeling asset price dynamics with greater precision is crucial for financial forecasting, especially when capturing sudden and significant fluctuations in returns. Traditional models, such as the Black-Scholes, rely on continuous diffusion processes and fail to represent the abrupt market movements observed in practice. To address this, [3] proposed a jump-diffusion model that integrates both continuous Brownian motion and a compound Poisson process to account for discontinuities in asset prices.

Let S_t denote the asset price at time t. The log-return dynamic under Kou's model is described as follows:

$$\frac{dS_t}{S_{t^-}} = \mu dt + \sigma dW_t + d\left(\sum_{k=1}^{N_t}(X_k - 1)\right) \tag{1}$$

where
S_t : asset price at time t
S_{t^-} : asset price immediately before a jump occurs at time t
μ : instantaneous drift rate of the asset return
σ : volatility parameter of the diffusion component
W_t : a standard Brownian motions
N_t : a Poisson process with parameter λ, representing the arrival rate of jumps
λ : expected number of jumps per unit time
X_k : a sequence of independent identically distributed (i.i.d.) random variables

Furthermore, let $Y_k = \ln(X_k)$ is the logarithmic jump size that assumed to follow an asymmetric double exponential (ADE) distribution with probability density function:

$$f_{Y_k}(y) = p\eta^+ e^{-\eta^+ y} 1_{\{y\geq 0\}} + (1-p)\eta^- e^{\eta^- y} 1_{\{y<0\}} \tag{2}$$

with η^+ and η^- are the rate parameters governing the magnitudes of positive and negative jumps, p and $(1-p)$ are the probabilities of positive and negative jumps. This distribution enables the model to capture the empirical feature of asset returns, such as leptokurtosis and skewness, which are commonly observed in financial time series [5].

The Kou jump-diffusion model has several advantages over its predecessors. Notably, it allows for closed-form solutions for pricing European options, improves modeling of extreme events, and accommodates asymmetries in jump direction and magnitude [12]. This makes it particularly suitable for option valuation, risk management, and asset-liability management applications.

3. DYNAMIC CPPI (D-CPPI) HEDGING STRATEGY

In portfolio management, ensuring downside protection while capturing upside potential is a core objective. One widely adopted approach that facilitates this balance is the Constant Proportion Portfolio Insurance (CPPI) strategy. The dynamic CPPI (D-CPPI) model builds upon the classical

CPPI by adjusting the risk multiplier in response to changing market conditions or investor preferences, thus providing greater flexibility in risk management [3].

In the CPPI strategy, the risk multiplier is constant and is denoted as m. In contrast, the D-CPPI strategy employs a dynamic risk multiplier, denoted as m_t. The risk multiplier value m_t according to [16] is defined as:

$$m_t = m_{t-1} + a \ln\left(\frac{S_t}{S_{t-1}}\right), t = 2,3 \ldots \tag{3}$$

where
m_t : dynamic risk multiplier at time t
S_t : the price of the risky asset at time t
a : an amplifier determined by investor's preferred risk

Eq. (5) shows that if the price of a risky asset increases, i.e., $S_t > S_{t-1}$, then $\ln(S_t > S_{t-1})$ will be positive. As a result, the dynamic risk multiplier, m_t, increases proportionally to that of the amplifier a. Instead, when the price of a risky asset decreases, i.e., $S_t < S_{t-1}$, the logarithmic term becomes negative, causing the dynamic risk multiplier m_t to decrease. This adjustment allows the portfolio exposure to respond to market movements, potentially enhancing upside participation while controlling downside risk. While, the greater the value of a, the more sensitive the investor is to changes in the return and risk of the risky asset [7]. Furthermore, the portfolio value at time *t* is given by Eq. (6).

$$\begin{gathered} V_t = E_t + R_t + O_t \\ C_t = V_t - F_t \\ E_t = m_t C_t \end{gathered} \tag{4}$$

where
V_t : portfolio value at time t
E_t : the risky asset value at time t
R_t : the riskless asset value at time t
C_t : the cushion value at time t
F_t : the floor value at time t
O_t : accounts for transaction costs

The initial floor is given by $F_0 = \lambda V_0$, where λ represents the proportion of the portfolio value that is guaranteed. The floor then grows with the riskless interest rate, as described by equation $F_t = e^{it} F_0$, or equivalently $F_t = e^{i} F_{t-1}$, where i is the riskless interest rate. It is important to note that the floor represents the guaranteed minimum benefit. The balancing of portfolio value continues until finished. Moreover, the portfolio value at time grows with return of risky asset and interest rate of riskless asset. Thus, the portfolio evolves as follows:

$$V_{t+1} = E_t e^{r_t} + R_t e^{i} \tag{5}$$

and

$$r_t = \ln\left(\frac{S_t}{S_{t-1}}\right) \tag{6}$$

where
V_{t+1} : portfolio value at time $t+1$
r_t : return on the risky asset at time t
i_t : return on the riskless asset at time t

However, a potential drawback emerges during abrupt negative market movements or frequent rebalancing. Thus, the portfolio value may fall below the floor or the guaranteed minimum benefit due to fluctuations in the price of the risky asset [15]. This condition is referred to the gap risk, $V_t <$

F_t [1], [2]. Consequently, hedging is necessary to mitigate this risk. In this condition, [10] selecting and adjusting the initial dynamic risk multiplier to hedging by:

$$m_0 = \eta \frac{(\mu - i)}{\sigma^2} \tag{7}$$

where
$\eta \in (0,1)$: a scaling parameter
μ : the expected return of the risky asset
i : the riskless interest rate
σ^2 : the variance of the risky asset's returns

By dynamically adjusting m_t and considering trading frictions, the D-CPPI strategy maintains capital protection against market drops while minimizing gap risk. Therefore, the D-CPPI strategy provides a suitable framework for insurers managing insurance portfolios with embedded capital protection features.

4. SIMULATION ANALYSIS

4.1 DATA

The data used as the risky asset consists of the closing prices of the S&P 500 index from 2006 to 2010. The price movements are analyzed on a daily, weekly, and monthly basis. The corresponding graphs are presented as follows:

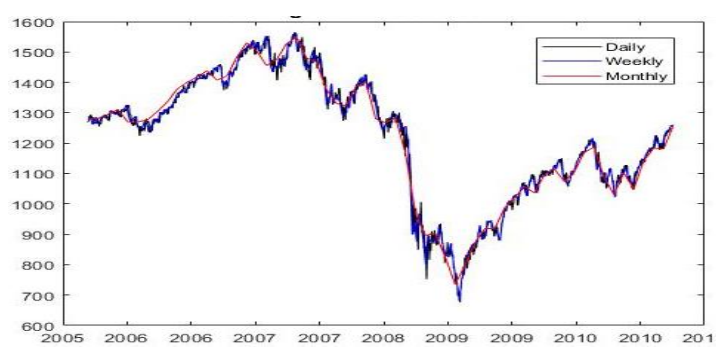


Figure 4.1. Daily, weekly, and monthly closing prices of the S&P 500 index during 2006-2010

The dataset consists of the daily closing prices of the S&P 500 index from 2006 to 2010. To evaluate the forecasting performance of the proposed model, the data are divided into training data (period 2006–2009) and testing data (period 2010). The training data are used to estimate the parameters of the Kou jump-diffusion model, while the testing data are employed to assess forecasting accuracy and the effectiveness of the proposed hedging strategy.

Period of the training data includes both normal market conditions and the 2008 global financial crisis, thereby providing a rich dataset for capturing volatility dynamics and jump behavior. This is particularly important for estimating the parameters of the Kou jump-diffusion model, which is designed to accommodate sudden and significant market movements. The mean and standard deviation values computed from the historical training data are presented as follows:

Table 4.1. Mean and standard deviation of the training data.

	Daily	Weekly	Monthly
Mean	1,283.6	1,238.4	1,240
Standard deviation	224.4758	224.4815	225.3982

Table 4.1 presents the descriptive statistics of the training dataset at different observation frequencies. The mean values indicate that the average level of the S&P 500 index during the training period remains relatively consistent across daily, weekly, and monthly observations, ranging from approximately 1,238 to 1,284. This suggests that changing the sampling frequency does not substantially alter the overall price level of the index. Returns were calculated from the S&P 500 index prices in the training period (2006–2009) at daily, weekly, and monthly frequencies using Eq. (6). The resulting return series are presented in **Figure 4.2.**

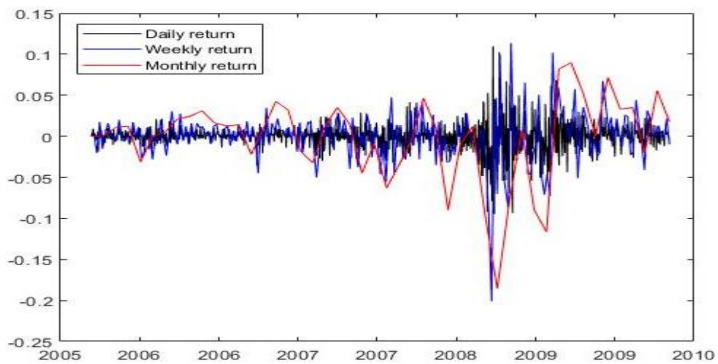


Figure 4.2. Daily, weekly, and monthly returns of the S&P 500 index during 2006–2009

Figure 4.2 shows that the return series fluctuate around zero but exhibit several extreme positive and negative observations, particularly during the 2008–2009 financial crisis. The daily returns display the highest variability, whereas the weekly and monthly returns appear smoother because of temporal aggregation. Nevertheless, substantial fluctuations remain evident across all frequencies.

The clustering of large return movements and the presence of abrupt market declines indicate that the return process cannot be adequately characterized by a simple diffusion model. These empirical features motivate the use of the Kou jump-diffusion model, which explicitly incorporates discontinuous jumps and asymmetric return behavior. Such characteristics are particularly relevant for portfolio insurance applications, as extreme market movements may increase gap risk and affect the performance of the D-CPPI strategy. To further examine the distributional characteristics of the return series, descriptive statistics including the mean, standard deviation, skewness, kurtosis, and selected percentiles were calculated. In this study, kurtosis values were computed using the MATLAB kurtosis function. A kurtosis value greater than three indicates a leptokurtic distribution with heavier tails and a higher probability of extreme observations than the normal distribution. The descriptive statistics are presented in **Table 4.2**.

Table 4.2. Descriptive statistics of the return data.

	Daily	Weekly	Monthly
Mean	-0.000128	-0.000680	-0.0029
Standard deviation	0.0166	0.0321	0.0521
Skewness	-0.2158	-0.9903	-1.1776
Kurtosis	11.2846	10.9280	5.0448
90^{th} percentile	0.0149	0.0280	0.0501
10^{th} percentile	-0.0170	-0.0368	-0.0860

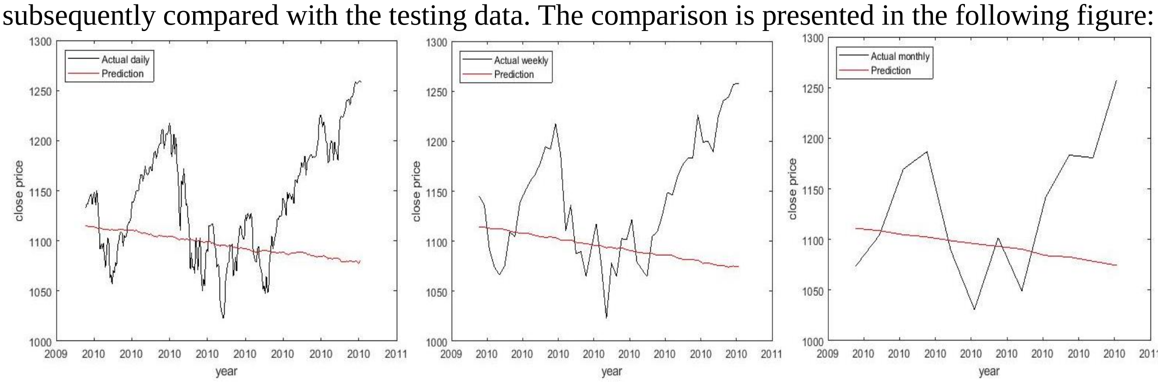
Table 4.2 shows that the return distributions exhibit negative skewness across all observation frequencies, indicating a tendency toward larger downside movements. Furthermore, the kurtosis values substantially exceed the benchmark value of three associated with the normal distribution. This leptokurtic behavior suggests the presence of heavy tails and a greater likelihood of extreme market movements. These findings support the use of the Kou jump-diffusion model, which is capable of capturing asymmetric return distributions and discontinuous price changes that cannot be adequately represented by a standard diffusion model.

Accordingly, future index prices are forecasted using the Kou model, which explicitly incorporates jump components in the return process. A kurtosis value exceeding three indicates a heavy-tailed distribution and suggests the presence of extreme observations that may be associated with jump behaviour [9]. To identify unusually large return movements, this study employs the 10th and 90th percentile thresholds. Returns below the 10th percentile are classified as extreme negative movements, whereas returns above the 90th percentile are classified as extreme positive movements.

Based on these thresholds, daily returns smaller than −0.0170 or greater than 0.0149 are identified as extreme observations. Similarly, weekly returns below −0.0368 or above 0.0280, and monthly returns below −0.0860 or above 0.0501, are classified as extreme return movements that may indicate jump events.

4.2 KOU JUMP PREDICTION MODEL

Predicted index values were generated using 10,000 Monte Carlo simulations based on the estimated Kou jump-diffusion parameters. Then, the predicted index values were generated and subsequently compared with the testing data. The comparison is presented in the following figure:



(a) (b) (c)

Figure 4.3. Testing data vs. predicted data, observed on a (a) daily, (b) weekly, and (c) monthly basis

Figure 4.3 compares the actual S&P 500 index values with the forecasts generated by the Kou jump-diffusion model during the out-of-sample period in 2010. The predicted series exhibit smoother trajectories than the observed index values, reflecting the inherent difficulty of forecasting the timing and magnitude of future market shocks. While the actual index displays substantial fluctuations, the forecasts primarily capture the overall trend and average level of the market.

Visual inspection indicates that the discrepancy between the actual and predicted values increases during periods of pronounced market movements. Nevertheless, the forecasts remain within a reasonable range of the observed index levels across all observation frequencies. The historical return series used for model estimation exhibits negative average returns across daily, weekly, and monthly frequencies, largely due to the inclusion of the 2008 global financial crisis, which exerted substantial downward pressure on equity prices. Consistent with these market conditions, the forecasted returns derived from the predicted index values also remain negative across all observation frequencies, indicating that the model preserves the directional characteristics of the underlying market environment. However, the quality of the forecasts cannot be assessed solely from the sign of the predicted returns. Therefore, forecasting performance is further evaluated using quantitative accuracy measures, namely Mean Absolute Percentage Error (MAPE), Root Mean Squared Error (RMSE), and Mean Absolute Error (MAE). The corresponding results are reported in **Table 4.3**.

Table 4.3. MAPE values.

	Daily	Weekly	Monthly
MAPE	0.0476	0.0504	0.0536
RMSE	71.297	77.754	78.458
MAE	55.788	59.429	62.374

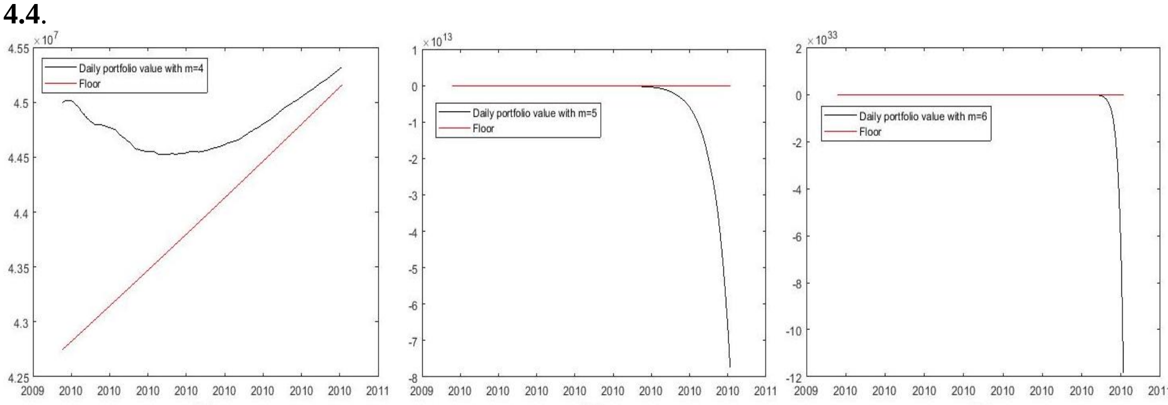
The forecasting performance of the Kou jump-diffusion model is summarized in Table 4.3. The results indicate that the daily model achieves the lowest forecasting errors, with a MAPE of 4.76%, RMSE of 71.297, and MAE of 55.788. The weekly and monthly models yield slightly higher error values, although the differences across frequencies remain relatively modest.

The MAPE values range from 4.76% to 5.36% across all observation frequencies, indicating that the forecasted index values remain close to the corresponding observed values during the out-of-sample period. These results demonstrate that the Kou jump-diffusion model provides satisfactory forecasting performance despite the presence of market volatility, asymmetric shocks, and discontinuous price movements. The daily specification exhibits the highest predictive accuracy among the three frequencies, which may be associated with the larger number of observations available for parameter estimation and the resulting ability of the model to better capture short-term market dynamics.

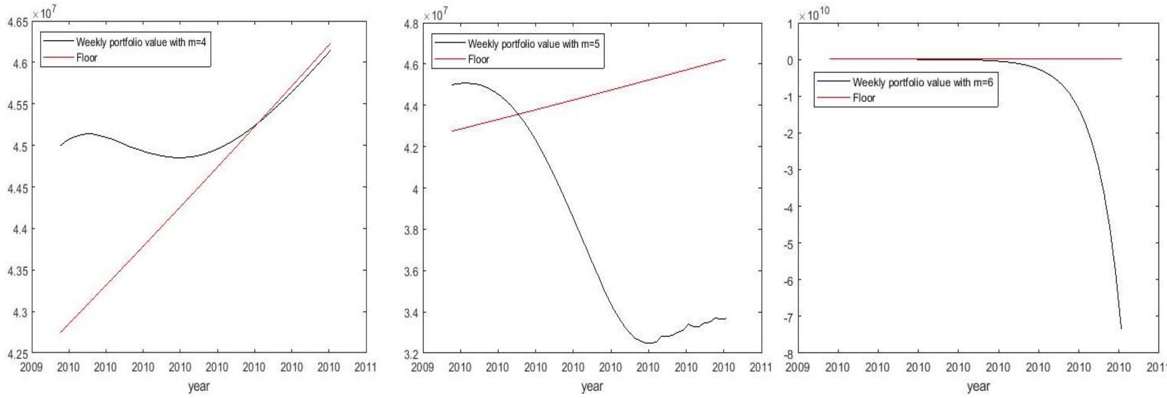
Overall, the forecasting results support the suitability of the Kou jump-diffusion model as the predictive component of the proposed insurance portfolio valuation framework.

4.2 VALUATION OF PORTFOLIO VALUE

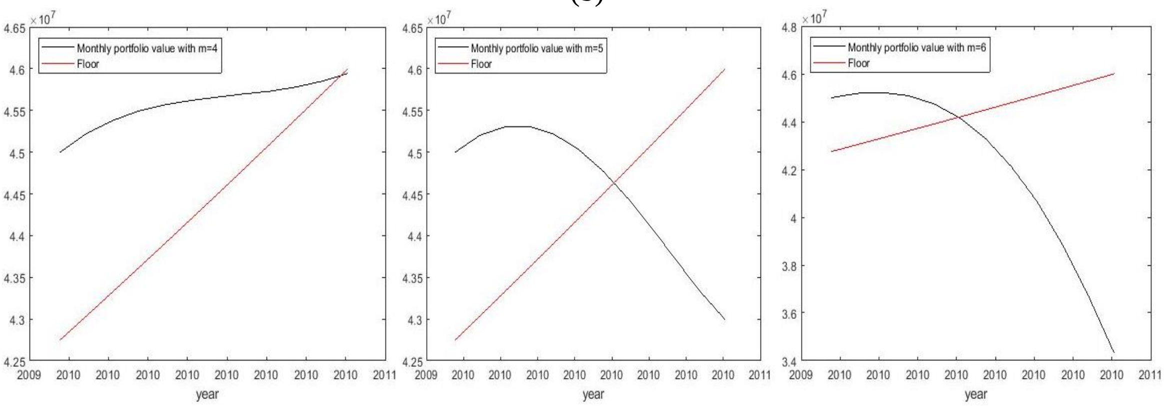
A portfolio valuation was carried out using the D-CPPI (Dynamic Constant Proportion Portfolio Insurance) strategy, based on the predicted data previously obtained using the Kou Jump model. In this simulation, it is assumed that the initial premium is USD 50 million with an acquisition cost of 10%, resulting in an initial portfolio value of USD 45 million. Additionally, an asset allocation cost of 22% per transaction is considered, along with an annual riskless rate of $i = 8\%$, and $\lambda = 95\%$. Furthermore, arbitrary initial risk multipliers were selected, for example $m_0 = 4$, $m_0 = 5$, and $m_0 = 6$. The resulting portfolio values compared to the corresponding floor values are presented in **Figure 4.4**.



(a)



(b)



(c)

Figure 4.4. Portfolio value vs. the floor, observed on a (a) daily, (b) weekly, and (c) monthly with initial risk multipliers $m_0 = 4$, $m_0 = 5$, and $m_0 = 6$ (from left to right)

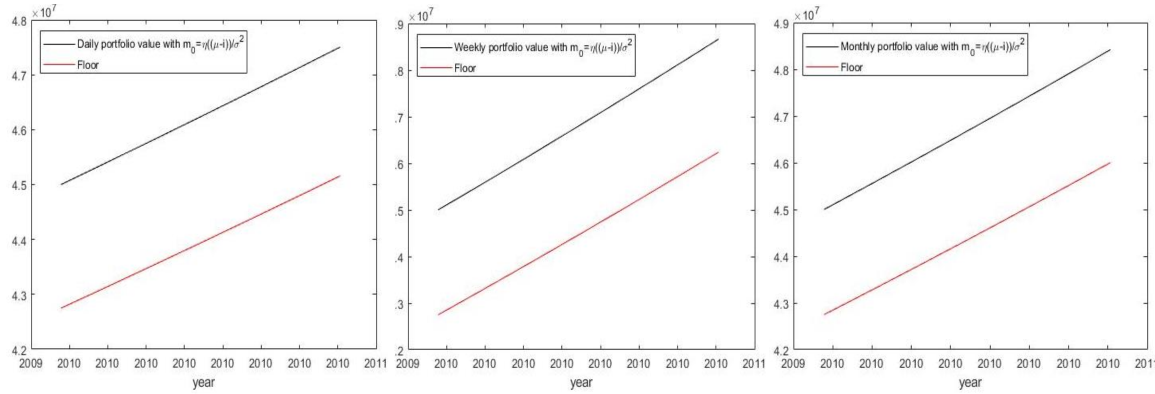
Figure 4.4 presents the portfolio dynamics generated by the D-CPPI strategy using arbitrarily selected initial multipliers ($m_0 = 4$, $m_0 = 5$, and $m_0 = 6$). The results reveal that portfolio performance is highly sensitive to the choice of the initial multiplier. For $m_0 = 4$, the portfolio generally remains close to or above the floor value throughout most of the investment horizon. However, in the weekly and monthly observations, the portfolio approaches the floor toward the end of the period, indicating a potential vulnerability to adverse market movements. When the multiplier is increased to $m_0 = 5$, the portfolio exhibits larger fluctuations and, in several cases, falls below the floor level. This effect becomes even more pronounced for $m_0 = 6$, where the portfolio experiences substantial losses and significant deviations from the protection floor.

These results demonstrate that selecting the initial multiplier arbitrarily may lead to insufficient protection against downside risk. In particular, excessively large multiplier values increase exposure to the risky asset and amplify the impact of unfavorable market movements, thereby increasing the likelihood of floor violations [8]. Consequently, a more systematic approach for determining the initial multiplier is required to improve the effectiveness of the D-CPPI strategy and mitigate gap risk.

To address this limitation, [10] proposed a volatility-adjusted multiplier defined as $m_0 = \eta \frac{(\mu - i)}{\sigma^2}$ in Eq. (7). However, the historical return data used for parameter estimation (2006–2009) exhibit negative average returns across daily, weekly, and monthly frequencies. This behavior is largely attributable to the inclusion of the 2008 global financial crisis, which generated substantial losses in the S&P 500 index. As a result, the estimated excess return becomes negative, yielding negative values of the volatility-adjusted multiplier. A negative multiplier implies a negative allocation to the risky asset, corresponding to a short position. However, the conventional CPPI framework assumes that exposure to the risky asset remains non-negative and does not exceed total portfolio wealth. Therefore, to maintain consistency with the portfolio insurance mechanism, the multiplier is constrained as

$$m_0 = max\left(0, \eta \, \frac{(\mu - i)}{\sigma^2}\right) \tag{8}$$

Under this specification, the portfolio exposure satisfies $0 \leq E_t \leq V_t$ condition. thereby preventing short positions and preserving the standard CPPI allocation structure. The resulting portfolio-to-floor dynamics obtained using Eq. (8) with $\eta = 0.5$ are presented in **Figure 4.5**.



(a) (b) (c)

Figure 4.5. Portfolio value vs. the floor, observed on a (a) daily, (b) weekly, and (c) monthly basis with initial risk multiplier $m_0 = \eta \, \frac{(\mu - i)}{\sigma^2}$

Figure 4.5 illustrates the portfolio dynamics obtained using the constrained volatility-adjusted multiplier. The results indicate that the portfolio value remains consistently above the protection floor across daily, weekly, and monthly observation frequencies. Unlike the portfolios generated using arbitrarily selected multipliers, no floor violations are observed throughout the investment horizon. This finding suggests that the proposed multiplier constraint effectively mitigates gap risk and enhances downside protection.

It should be noted that the estimated excess returns are negative during the estimation period, resulting in a theoretical multiplier below zero. After imposing the non-negativity constraint, the multiplier becomes zero, implying no allocation to the risky asset. Consequently, the portfolio is invested entirely in the riskless asset, producing a smooth growth pattern and ensuring continuous preservation of the protection floor. While this conservative allocation substantially reduces downside risk, it also limits participation in potential market gains. To provide a quantitative comparison between the arbitrarily selected multipliers and the constrained volatility-adjusted multiplier, the final portfolio values and their corresponding floor levels at the end of the investment horizon are reported in **Table 4.4**.

Table 4.4. Final portfolio,V_n, and floor values, F_n, under different initial multipliers and hedging conditions

	Daily		Weekly		Monthly	
	V_n	F_n	V_n	F_n	V_n	F_n
$m_0 = 4$	$4.5322e^7$	$4.5158e^7$	$4.6153e^7$	$4.6239e^7$	$4.5944e^7$	$4.6003e^7$
$m_0 = 5$	$-7.7404e^{13}$	$4.5158e^7$	$3.3683e^7$	$4.6239e^7$	$4.2987e^7$	$4.6003e^7$
$m_0 = 6$	$-1.1830e^{34}$	$4.5158e^7$	$-7.3624e^{10}$	$4.6239e^7$	$3.4308e^7$	$4.6003e^7$

$m_0 = \eta \dfrac{(\mu - i)}{\sigma^2}$	$4.7504e^7$	$4.5158e^7$	$\mathbf{4.8664e^7}$	$4.6239e^7$	$4.8418e^7$	$4.6003e^7$

Table 4.4 confirms the observations obtained from **Figures 4.4** and **4.5**. For the arbitrarily selected multipliers, portfolio performance deteriorates as the multiplier increases. Although the portfolio generated with $m_0 = 4$ remains relatively close to the protection floor, the weekly and monthly portfolios eventually fall below their corresponding floor values. The deterioration becomes substantially more severe for $m_0 = 5$ and $m_0 = 6$, where several portfolios experience significant losses and large deviations from the protection floor. In particular, the extremely negative portfolio values observed under the daily specification indicate the instability associated with excessive exposure to the risky asset during adverse market conditions.

By contrast, the constrained volatility-adjusted multiplier consistently maintains portfolio values above their corresponding floor levels across daily, weekly, and monthly frequencies. The resulting final portfolio values are not only protected from floor violations but also exceed those obtained under the arbitrarily selected multipliers. These findings demonstrate that the proposed multiplier adjustment effectively mitigates gap risk and provides a more robust portfolio insurance mechanism. This characteristic is particularly important for insurance portfolio valuation, where capital preservation and downside protection are often prioritized over aggressive return generation.

Overall, the empirical results demonstrate that the integration of Kou jump-diffusion forecasting and the D-CPPI strategy provides an effective framework for insurance portfolio valuation under volatile market conditions. The forecasting component captures the asymmetric and discontinuous behavior of asset prices, while the portfolio insurance mechanism adjusts portfolio exposure according to prevailing market conditions. The findings indicate that the use of a volatility-adjusted multiplier substantially improves downside protection and reduces the likelihood of floor violations compared with arbitrarily selected multipliers. Consequently, the proposed framework offers a practical approach for preserving portfolio value while mitigating gap risk in insurance portfolio management.

The results also highlight the sensitivity of D-CPPI based strategies to market conditions. During periods characterized by negative expected returns, the volatility-adjusted multiplier may converge to zero after applying the non-negativity constraint, resulting in a fully riskless allocation. Although this conservative positioning effectively preserves the protection floor, it may reduce participation in potential market recoveries. Therefore, the proposed approach should be viewed as a risk-management mechanism prioritizing capital preservation rather than return maximization. This characteristic is particularly relevant for insurance portfolio valuation, where downside protection and capital preservation are fundamental objectives.

5. CONCLUSION

This study proposed an insurance portfolio valuation framework that integrates the Kou jump-diffusion forecasting model with a Dynamic Constant Proportion Portfolio Insurance (D-CPPI) strategy. The framework was developed to capture discontinuous asset-price movements while simultaneously providing downside protection through a portfolio insurance mechanism. The main conclusions of this study are as follows:

1. The Kou jump-diffusion model demonstrated satisfactory forecasting performance across daily, weekly, and monthly observation frequencies, achieving MAPE values of 4.76%, 5.04%, and 5.36%, respectively. These results indicate that the model is capable of representing the overall dynamics of the S&P 500 index in the presence of volatility and jump events.
2. The performance of the D-CPPI strategy was found to be highly sensitive to the choice of the initial multiplier. Arbitrarily selected multipliers increased exposure to risky assets and resulted in significant portfolio losses and floor violations under adverse market conditions.

3. The proposed volatility-adjusted multiplier provided a systematic mechanism for determining portfolio exposure. Because the estimated excess returns were negative during the study period, the non-negativity constraint resulted in a zero multiplier and a fully risk-free allocation. Under this setting, the resulting portfolios consistently remained above their corresponding floor levels across all observation frequencies, thereby eliminating floor violations observed under arbitrarily selected multipliers.
4. The integration of Kou jump-diffusion forecasting and the constrained volatility-adjusted D-CPPI strategy effectively mitigated gap risk and improved downside protection. Therefore, the proposed framework offers a practical and effective approach for insurance portfolio valuation by combining market forecasting capability with enhanced downside-risk protection in volatile financial market.
5. A limitation of this study is that the estimation period includes the 2008 global financial crisis, which generated negative expected returns and resulted in a highly conservative portfolio allocation. Future research may explore alternative multiplier adjustment mechanisms, different estimation periods, or broader asset classes to further evaluate the robustness and applicability of the proposed framework.

CONFLICT OF INTEREST

The authors declare that there is no conflict of interest

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