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Connectivity of Non-Coprime Graph over Dihedral Group

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Abstract

Let Γ be the non-coprime graph of the dihedral group of order $2n$. This research investigates the connectivity properties of Γ particularly its Eulerian, semi-Eulerian, Hamiltonian, and planar properties. The research begins by examining several properties related to the dihedral group, and then identifying the connectivity patterns that are formed in the non-coprime graph through the analysis of the order of elements in the group. These patterns then serve as the basis for determining its connectivity properties. The results show that Γ is Eulerian when $n = 2^k$, where $k \geq 2$, and semi-Eulerian if and only if $n = 2p$, where p is an odd prime. Furthermore, the graph is Hamiltonian for $n = 2^k$ or $n = 2p$, where p is an odd prime. In addition, the graph is planar if and only if $n = 2$ or $n = 3$. These results indicate that the factorization form of n strongly influences the connectivity structure and graph properties of the non-coprime graph over the dihedral group.

Keywords: Non-coprime graph, dihedral graph, Eulerian graph, Hamiltonian graph, Planar graph.

1. INTRODUCTION AND PRELIMINARIES

Graph theory is one of the branches of mathematics used to represent relationships among objects in the form of vertices (nodes) and edges. This concept is widely applied to simplify complex problems into models that are easier to analyze, particularly in studying the structure and connectivity of a system. Along with its development, graph theory has also been utilized as a representation of groups, so that concepts in group theory, which are abstract in nature, can be understood more deeply and visualized more easily.

One of the earliest forms of group representation through graphs is the prime graph, also known as the Gruenberg–Kegel graph. The prime graph of a finite group G is defined as a graph whose vertex set consists of the prime divisors of $|G|$, where two distinct vertices p and q are adjacent if and only if G contains an element of order pq [6]. This concept was introduced in the 1970s to study cohomological problems related to integral representations of finite groups, and has since become one of the most studied graphs associated with finite groups [6]. Research on prime graphs has largely focused on simple and solvable groups, including the purely graph-theoretical characterization of prime graphs for solvable groups, as well as the classification of connected components of prime graphs for various finite groups [20],[10].

Building on the idea of associating primes with group structure, another representation that has been widely studied is the coprime graph. The coprime graph of a finite group G, denoted $\Gamma(G)$, is defined with the vertex set being all elements of G, where two distinct vertices x and y are adjacent if and only if $gcd(|x|,|y|) = 1$ [11]. This concept was further developed by examining various graph parameters such as diameter, girth, independence number, and clique number for several classes of finite groups, including cyclic groups, dihedral groups, and generalized quaternion groups [17],[19],[14]. Other studies have also explored the classification of finite groups whose coprime graphs are split, threshold, or chordal graphs [3], as well as the coprime graph of subgroups of a group, which considers the orders of subgroups instead of elements [7]. Research related to connectivity and connectivity indices of coprime graph over some group can be found in [16], [15].

In contrast to the coprime graph, the complement of this concept gives rise to the non-coprime graph, which was introduced and developed in [12] to investigate the general properties and structures of such graphs for various types of finite groups. Subsequent studies expanding upon this research include [2], which examined the Cartesian product of non-coprime graphs over finite groups, while properties related to their graph energy were discussed in [8]. Furthermore, a study generalizing this type of graph was proposed in [9], where connectivity was established by incorporating subgroups within the group.

However, existing literature still presents several limitations, as previous work did not specifically examine the structure of non-coprime graphs over dihedral groups. In addition, previous discussions mainly focused on specific graph parameters such as the clique number, chromatic number, independence number, clique covering number, and diameter in certain special cases. Therefore, this research focuses on investigating graph connectivity restricted to Eulerian and Hamiltonian properties for D_{2n} and D_{2p}, where n is a natural number and p is an odd prime number, as well as the planarity property for D_{2n} and D_{2p}.

In this research, the authors employed a literature study methodology. The literature sources were gathered from reference books and academic journals relevant to the research topic, specifically focusing on the foundational concepts of non-coprime graphs, group theory, the dihedral group D_{2n}, and the connectivity of non-coprime graphs over the dihedral group D_{2n}. Prior to presenting the main results, several preliminary definitions and essential theorems regarding non-coprime graphs and dihedral groups are introduced.

Definition 1.1. [13] *A graph G consists of a nonempty finite vertex set $(V(G))$ and a finite edge set $(E(G))$, such that:*

1. *Each edge in $E(G)$connects exactly two distinct vertices in G.*
2. *Any two vertices in $V(G)$are connected by at most one edge or are not connected at all.*

Definition 1.2. [13] *Let Gbe a graph and let u, v be vertices in G. A vertex u is said to be* ***connected*** *to v if there exists a path joining them. If every pair of vertices in $G$is connected, then G is called a* ***connected graph****.*

Definition 1.3. [13] *A* ***walk*** *in a graph G is a sequence of vertices and edges in G of the form*

$$v_1 e_1 v_2 e_2 \cdots v_k,$$

where $k \geq 1$ and each edge e_i is incident with the vertices v_i and v_{i+1}. The length of a walk is the number of edges contained in the sequence. Such a walk is also often denoted by $v_1 - v_k$.

A walk that does not repeat any edge is called a **trail**, whereas a walk that does not repeat any vertex is called a **path** [13]. In addition to restrictions on repeating edges and vertices, a walk can also be classified based on whether its initial and terminal vertices are the same. A walk that starts and ends at the same vertex is called a **closed walk**, while a closed walk that does not repeat any edge is called a **circuit**, and a circuit that does not repeat any vertex is called a **cycle** [13].

Definition 1.4. [13] *A connected graph G is called an Eulerian graph if there is a circuit in the graph that contains all edges in the graph G. A circuit that causes the graph to be Eulerian is called an Eulerian circuit.*

Theorem 1.5. [13] *A connected graph G is Eulerian if every vertex in G has even degree.*

Definition 1.6. [13]*Suppose G is a connected graph. G is called a semi-Eulerian graph if it contains an open Eulerian trail, that is, an open walk containing all edges with no repeated edges.*

Theorem 1.7. [13] *A connected graph G is semi-Eulerian if it contains exactly 2 vertices of odd degree.*

Definition 1.8. [13] *A connected graph G is called Hamiltonian if it contains a cycle (closed walk) that contains all the vertices. A cycle that causes the graph to be Hamiltonian is called a Hamiltonian Cycle.*

Theorem 1.9. [3] **(Dirac's Theorem)** *Let $G = (V, E)$ be a simple graph with n vertices. If every vertex in G has degree at least $\frac{n}{2}$, then G has a Hamiltonian cycle.*

Definition 1.10. [13] *A planar graph is a graph that can be drawn without crossing its edges, that is, a graph in which no two edges intersect except at one vertex where they are incident.*

Definition 1.11. [13] *Two graphs are called homeomorphic if they can be obtained from the same graph by adding vertices of degree two.*

Definition 1.12. [13]**(Kuratowski's Theorem)** *A graph is planar if and only if it does not contain a subgraph that is homeomorphic to K_5 atau $K_{3,3}$.*

Definition 1.13. [4] *The dihedral group denoted by D_{2n} is the set:*

$$D_{2n} = \{e, a, a^2, \dots, a^{n-1}, b, ab, a^2b, \dots, a^{n-1}b\}, n \geq 3.$$

In particular, all elements in $D_{2n}$of order greater than 2 are powers of a.

Definition 1.14. [12] *A non-coprime graph $\Gamma(G)$ of a finite group G Iis a graph with vertex set $G\backslash\{e\}$, and two distinct vertices u and v are adjacent if and only if $gcd\big(o(u), o(v)\big) \neq 1$.*

Theorem 1.15. [18] *Given a dihedral group*

$$D_{2n} = \langle \mathrm{a}, \mathrm{b} \mid \mathrm{a}^{\mathrm{n}} = \mathrm{e}, , \mathrm{b}^2 = \mathrm{e}, , \mathrm{bab} = \mathrm{a}^{-1} \rangle.$$

If 2^k, with $k \geq 2$ then the non-coprime graph of the dihedral group D_{2n} is the complete graph K_{2n-1}.

2. MAIN RESULTS

In this research, we will discuss the connectivity of non-coprime graphs built on the dihedral group D_{2n}. The discussion is focused on the study of Eulerian and Hamiltonian properties based on several forms of n, namely odd n, $n = 2^k$, and $n = 2^k q$, where q is an odd prime number. In addition, the planarity property of the non-coprime graph over the dihedral group D_{2n} is also investigated, which is determined by the value of n. Differences in the factorization form of n produce different connectivity structures, thereby affecting the Eulerian, Hamiltonia, and Planarity properties of the resulting graph.

For odd n, the non-coprime graph over the dihedral group D_{2n} is disconnected, Indeed, every reflection in D_{2n} has order 2, while every non-identity rotation has order dividing n. Since n is odd, the orders of a reflection and a non-identity rotation are relatively prime. Hence, there is no edge between any reflection and any non-identity rotation in the non-coprime graph. Consequently, the non-coprime graph over D_{2n} is disconnected. and therefore it is neither Eulerian and Hamiltonian. Hence, the subsequent discussion is focused on the case where n is even. The main result for the Eulerian property is given in the following theorem.

Theorem 2.1. *Given a dihedral group D_{2n}. If $n = 2^k$ where k is a natural number, then the non-coprime graph formed from the group D_{2n} is an Eulerian graph.*

Proof. Let $\mathrm{D}_{2\mathrm{n}}$ be a dihedral group with $\mathrm{n} = 2^k$, where $k \in \mathbb{N}$. Since all rotation elements have orders that are powers of 2 and every reflection element has order 2, every pair of non-identity elements has a nontrivial common divisor of orders. Hence, the non-coprime graph $\Gamma(D_{2n})$ is a complete graph.
Moreover, the graph has $2n - 1 = 2^{k+1} - 1$ vertices. Therefore, each vertex has degree

$$\deg(v) = \left(2^{k+1} - 1\right) - 1 = 2^{k+1} - 2,$$

which is even. Since $\Gamma(D_{2n})$ is connected and every vertex has even degree, therefore based on Theorem 1.5 it follows that $\Gamma(D_{2n})$ is Eulerian.

Theorem 2.2. *Given the dihedral group D_{2n}. The non-coprime graph formed from D_{2n} is semi-Eulerian if and only if $n = 2^k q$, with $k \in \mathbb{N}$ and $q = 3$.*

Proof. Let $\mathrm{D}_{2\mathrm{n}}$ be a dihedral group with $\mathrm{n} = 2^{\mathrm{k}}q$, where $\mathrm{k} \in \mathbb{N}$ and q is an odd prime. Suppose that the non-coprime graph $\Gamma(D_{2n})$ is semi-Eulerian. Since the rotation elements satisfy

$$o\left(a^i\right) = \frac{n}{\gcd(n,i)},$$

There exist exactly two rotation elements order q, namely a^{2^k} and $a^{2.2^k}$.
If $q = 3$, then these two vertices are adjacent to all vertices whose orders are divisible by 3. Hence,

$$deg\left(a^{2^k}\right) = deg\left(a^{2.2^k}\right) = 2^{k+1} - 1,$$

Which is odd. On other hand, every reflection element has order 2, and all remaining vertices have even degree. Therefore, the graph has exactly two vertices of odd degree and based on Theorem 1.7 its implying that $\Gamma(D_{2n})$ is semi-Eulerian.

Conversely, if $\Gamma(D_{2n})$ is semi-Eulerian, then based on Theorem 1.7 the graph must have exactly two odd degree vertices. This occurs precisely when $q = 3$, since only the two vertices corresponding to the elements of order 3 have odd degree, while all other vertices have even degree. Thus, the non-coprime graph D_{2n} is semi- Eulerian if and only if $n = 2^k.3$, where $k \in \mathbb{N}$.

Theorem 2.3. *Let $\Gamma_{D_{2n}}$ be the non-coprime graph over the dihedral group D_{2n}. If $n = 2^k$ or $n = 2^k q$, where $k \in \mathbb{N}$ and q is an odd prime number, then $\Gamma_{D_{2n}}$ is Hamiltonian*

Proof. Let D_{2n} be a dihedral group. If $n = 2^k$ for some $k \in \mathbb{N}$, then by Theorem 1.15., the non-coprime graph $\Gamma(D_{2n})$ is the complete graph K_{2n-1}. Since every complete graph with at least three vertices is Hamiltonian, it follows that $\Gamma(D_{2n})$ is Hamiltonian. Furthermore, let $n = 2^k q$, where $k \in \mathbb{N}$ and q is an odd prime. The graph $\Gamma(D_{2n})$ has $2n - 1$ vertices. Since every reflection element has order 2, each reflection is adjacent to all other reflections and to all rotations of even order. Consequently, every vertex has degree at least n, that is,

$$\delta\big(\Gamma(D_{2n})\big) \geq n.$$

Because

$$n > \frac{2n-1}{2},$$

We obtain

$$\delta\big(\Gamma(D_{2n})\big) \geq \frac{2n-1}{2}.$$

Therefore, By Dirac's Theorem, $\Gamma(D_{2n})$ is Hamilton Graph.

Example 2.4. Let the dihedral group D_4 and D_8 be defined as follows:

$$\mathrm{D}_4 = \{a, b, ab\} \text{ and } \mathrm{D}_8 = \{a, a^2, a^3, b, ab, a^2b, a^3b\}$$

Based on **Theorem 1.15.**, it is obtained that the non-coprime graphs formed from the dihedral groups D_4 and D_8 are respectively the complete graps K_3 and K_7, as shown in the following figures:

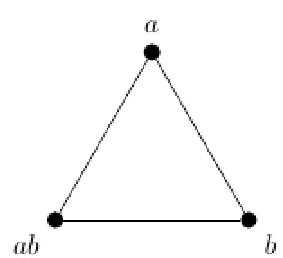

Figure 2.1. The non-coprime of the dihedral group D_4

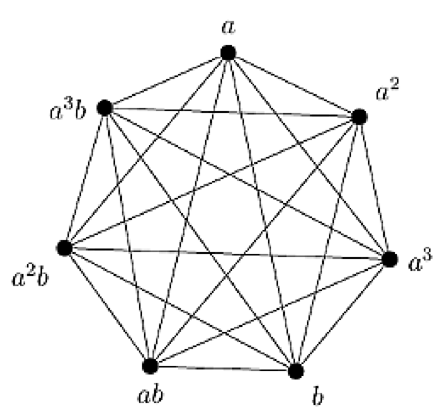

Figure 2.2. The non-coprime of the dihedral group D_8

Based on **Figure 2.1.**, it is clear that the complete graph K_3 is planar because it can be drawn in the plane without any intersecting edges and satisfies the requirements of Kuratowski's **Theorem 1.12.**. Therefore, $\Gamma(\mathrm{D}_4)$ is planar graph. Looking at **Figure 2.2.**, it can be seen that the complete graph K_7 contains the subgraph K_5, so based on **Theorem 1.12.**, $\Gamma(D_8)$ is not a planar graph.

Example 2.5. Let the dihedral group D_6 and D_{10} be defined as follows:

$$\mathrm{D}_6 = \{a, a^2, b, ab, a^2b\} \text{ and } \mathrm{D}_{10} = \{a, a^2, a^3, a^4, b, ab, a^2b, a^3b, a^4b\}$$

The non-coprime graph representation of D_6 and D_{10} is as follows:

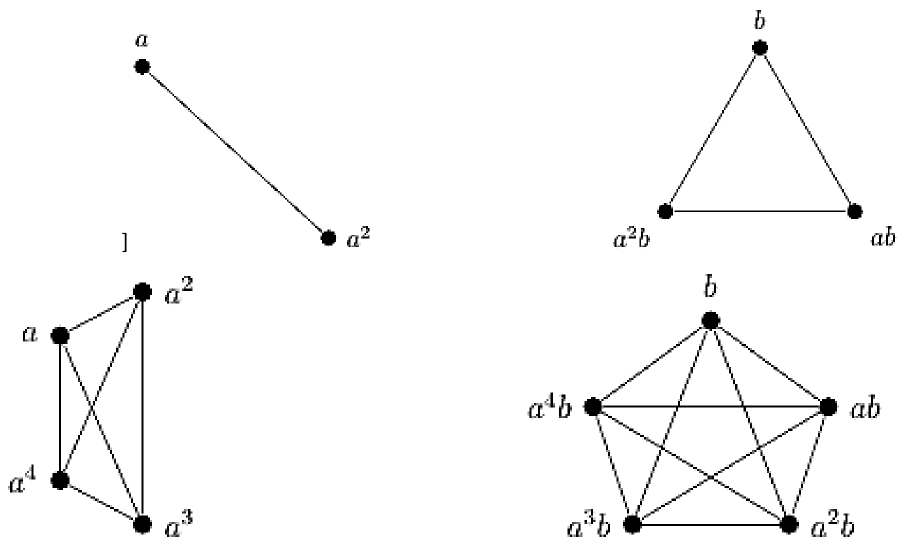

Figure 2.4. The non-coprime of the dihedral group D_{10}

Based on Figure 2.3., it can be seen that the graph $\Gamma(D_6)$ is divided into two disjoint graph components, namely, graphs K_2 and K_3, mathematically written as $D_6 \ = K_2 \cup K_3$. Because K_2 and K_3 are planar graphs, consequently, $\Gamma(D_6)$ is also planar. Furthermore, considering Figure 2.4., it can also be seen that $\Gamma(D_{10})$ is also divided into two disjoint graph components, namely the complete graphs K_4 and K_5, mathematically written as $D_{10} = K_4 \cup K_5$. Because $\Gamma(D_{10})$ directly contains the subgraph K_5, based on Theorem 2.14., it can be concluded that $\Gamma(D_{10})$ is not a planar graph.

Based on Examples 2.4. and 2.5., it can be analyzed that the non-coprime graphs over the dihedral groups D_4 and D_6 still form planar graphs, but when the value of n is enlarged in the case of D_8 and D_{10}, the non-coprime graphs formed contain non-planar subgraphs, so that based on Kuratowski's Theorem the graph is not planar. Thus, the planar property only appears at the values $n = 2$ and $n = 3$. The same pattern applies to every value of n, so this is an illustration of the following theorem:

Theorem 2.5. *Let D_{2n} be a dihedral group. The non-coprime graph formed from D_{2n} is planar if and only if $n = 2$ or $n = 3$.*
Proof. Let $D_{2\mathrm{n}}$ be a dihedral group. We prove the statement in both directions. Suppose that $n = 2$ or $n = 3$. For $n = 2$, the resulting non-coprime graph is isomorphic to the complete graph K_3,

and hence it is planar. For $n = 3$, the resulting graph is the union of two planar components, namely K_2 and K_3. Therefore, it is also planar.

Conversely, suppose that the non-coprime graph of $D_{2\mathrm{n}}$ is planar. We show that $n = 2$ or $n = 3$. If $n \geq 4$, all reflection elements have order 2. Since $gcd(2,2) = 2 \neq 1$, any two distinct reflection elements are adjacent. Thus, the reflection elements form a complete subgraph K_{n}. For $n \geq 5$, the graph contains K_5 as a subgraph. For $n = 4$, the resulting graph is K_7, which also contains K_5. By Theorem 2.14, any graph containing K_5 is non-planar. Hence, the non-coprime graph is non-planar for every $n \geq 4$. Therefore, if the non-coprime graph of $D_{2\mathrm{n}}$ is planar, then $n = 2$ or $n = 3$.

Combining the two implications, the non-coprime graph of the dihedral group D_{2n} is planar if and only if $n = 2$ or $n = 3$. ■

3. CONCLUSION

In this research, the connectivity properties of the non-coprime graph over the dihedral group D_{2n} have been investigated. The resukt show that the algebraic structure and factorization form of n determine the resulting graph properties. The non-coprime graph $\Gamma(D_{2n})$is Eulerian when $n = 2^k$, and semi-Eulerian if and only if $n = 2^k 3$, where $k \in \mathbb{N}$. Furthermore, the graph is Hamiltonian for $n = 2^k$ or $n = 2^k q$, with q is an odd prime. In terms of planarity, $\Gamma(D_{2n})$ is palanar if and only if $n = 2$ or $n = 3$. Thus, the connectivity and structure properties of the non-coprime graph are closely related to the order structure of the elements in the dihedral group.

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